Shifting Bottleneck Heuristic for Job Shop

The Shifting Bottleneck Heuristic

- successful heuristic to solve makespan minimization for job shop
- iterative heuristic
- determines in each iteration the schedule for one additional machine
- uses reoptimization to change already scheduled machines
- can be adapted to more general job shop problems
  - other objective functions
  - workcenters instead of machines
  - set-up times on machines
  - ...

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Basic Idea

- Notation: M set of all machines
- Given: fixed schedules for a subset $M^0 \subset M$ of machines (i.e. a selection of disjunctive arcs for cliques corresponding to these machines)
- Actions in one iteration:
  - select a machine k which has not been fixed (i.e. a machine from $M \setminus M^0$)
  - determine a schedule (selection) for machine k on the base of the fixed schedules for the machines in M^0
  - reschedule the machines from M^0 based on the other fixed schedules

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Selection of a machine

- Idea: Chose unscheduled machine which causes the most problems (bottleneck machine)
- Realization:
  - Calculate for each operation on an unscheduled machine the earliest possible starting time and the minimal delay between the end of the operation and the end of the complete schedule based on the fixed schedules on the machines in M^0 and the job orders
  - calculate for each unscheduled machine a schedule respecting these earliest release times and delays
  - chose a machine with maximal completion time and fix the schedule on this machine

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Technical realization

- Define graph $G' = (N, A')$:
  - N same node set as for the disjunctive graph
  - A' contains all conjunctive arcs and the disjunctive arcs corresponding to the selections on the machines in M^0
- $C_{max}(M^0)$ is the length of a critical path in G'

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Technical realization

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Comments:

- with respect to G' operations on machines from $M \setminus M^0$ may be processed in parallel
- $C_{max}(M^0)$ is the makespan of a corresponding schedule

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Technical realization (cont.)

- for an operation (i, j); $i \in M \setminus M^0$ let
  - r_{ij} be the length of the longest path from 0 to (i, j) (without p_{ij}) in G'
  - q_{ij} be the length of the longest path from (i, j) to $*$ (without p_{ij}) in G'

Comments:

- r_{ij} is the release time of (i, j) w.r.t. G'
- q_{ij} is the tail (minimal time till end) of (i, j) w.r.t. G'

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Technical realization (cont.)

- For each machine from $M \setminus M^0$ solve the nonpreemptive one-machine head-body-tail problem $1|r_j, d_j < 0|L_{max}$
- Result: values $f(i)$ for all $i \in M \setminus M^0$
- Action:
  - Chose machine k as the machine with the largest $f(i)$ value
  - schedule machine k according to the optimal schedule of the one-machine problem
  - add k to M^0 and the corresponding disjunctive arcs to G'
- $C_{max}(M^0 \cup k) \geq f(k)$

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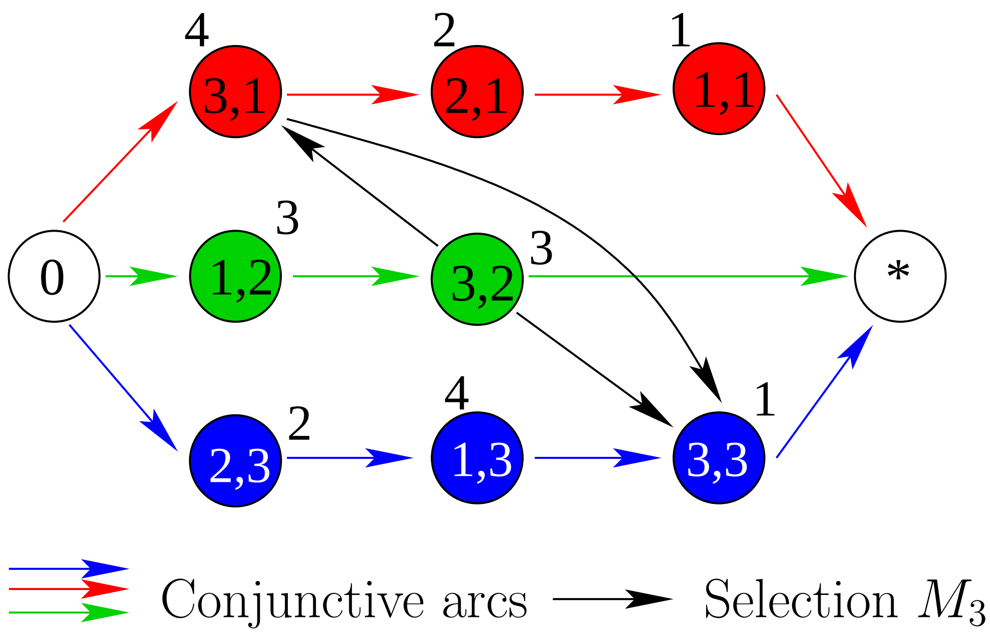
Technical realization - Example

- Given: $M^0 = \{M3\}$ and on M_3 the sequence $(3,2) \rightarrow (3,1) \rightarrow (3,3)$
- Graph G':



- $C_{max}(M^0) = 13$

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Technical realization - Example (cont.)

Machine M_1:

(i,j)	$(1,1)$	$(1,2)$	$(1,3)$
r_{ij}	12	0	2
q_{ij}	0	10	1
p_{ij}	1	3	4

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Technical realization - Example (cont.)

Machine M_1:

(i,j)	$(1,1)$	$(1,2)$	$(1,3)$
r_{ij}	12	0	2
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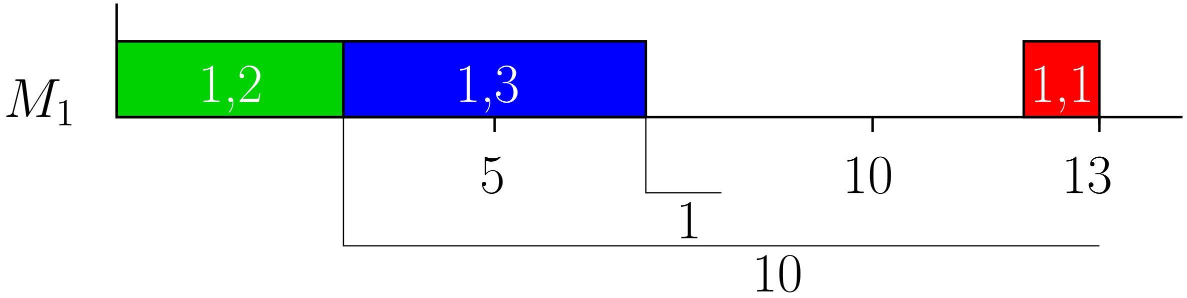

$$f(M_1) = 13$$

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Technical realization - Example (cont.)

Machine M_1:

(i,j)	$(1,1)$	$(1,2)$	$(1,3)$
r_{ij}	12	0	2
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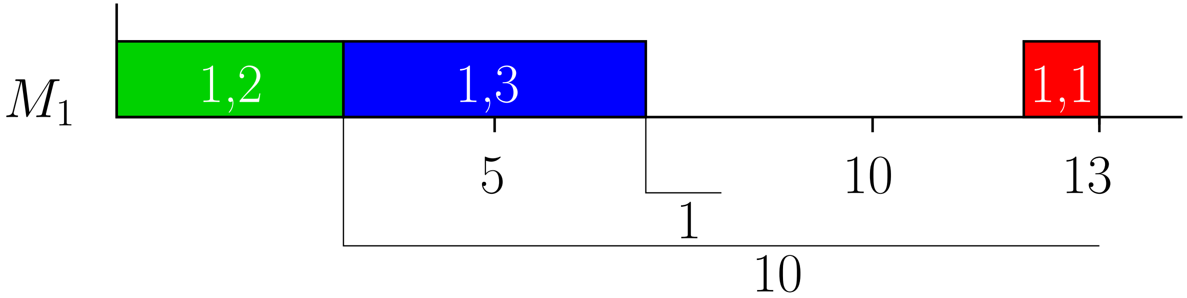

$f(M_1) = 13$

Machine M_2:

(i,j)	$(2,1)$	$(2,3)$
r_{ij}	10	0
q_{ij}	1	5
p_{ij}	2	2

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Technical realization - Example (cont.)

Machine M_1:

(i,j)	$(1,1)$	$(1,2)$	$(1,3)$
r_{ij}	12	0	2
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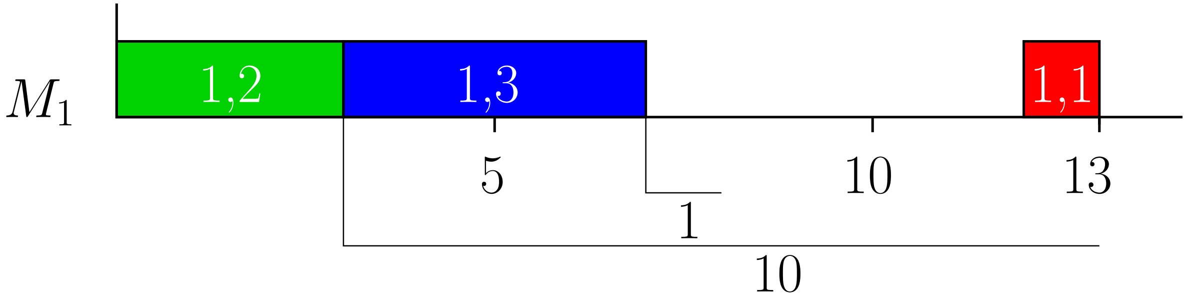

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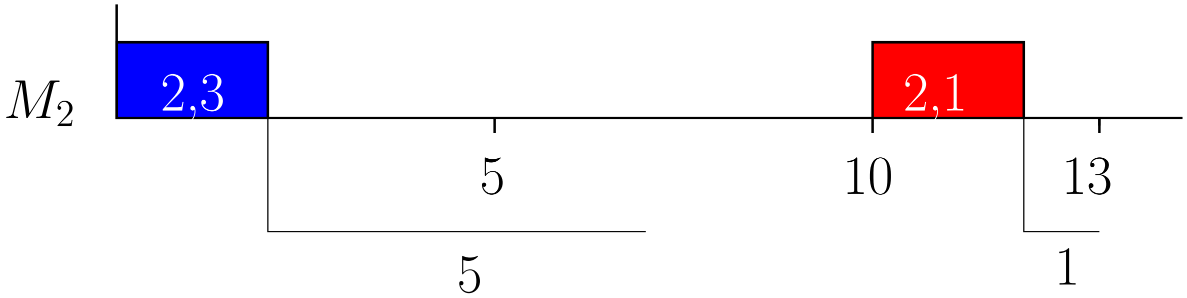

$$f(M_2) = 13$$

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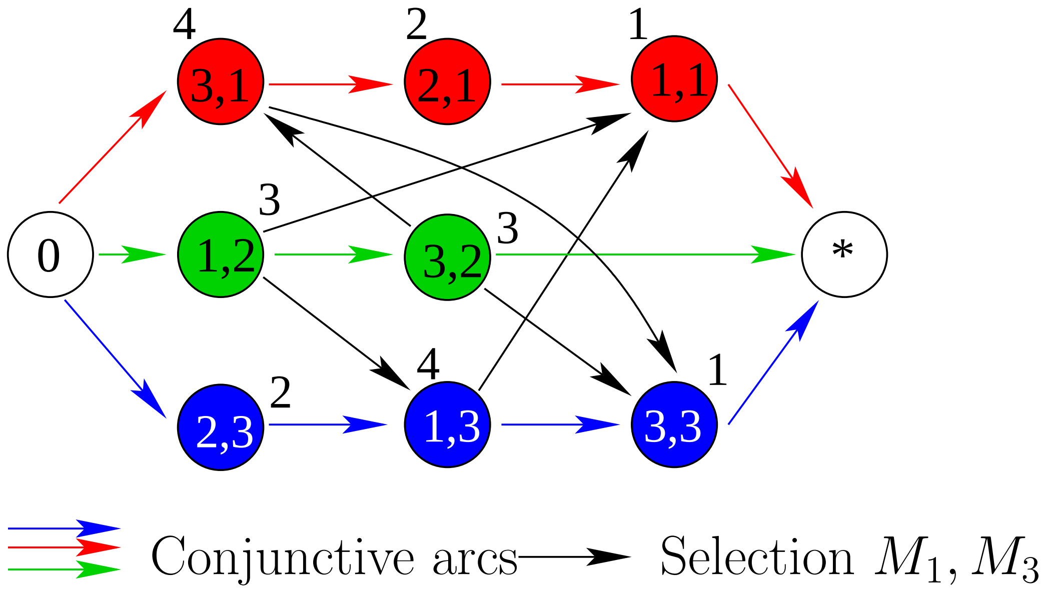
Technical realization - Example (cont.)

- Choose machine M_1 as the machine to fix the schedule:
  - add $(1,2) \longrightarrow (1,3) \longrightarrow (1,1)$ to G'
  - $M^0 = \{M_1, M_3\}$



- $C_{max}(M^0) = 13$

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<u>Reschedule Machines</u>

- try to reduce the makespan of the schedule for the machines in M^0
- Realization:
  - consider the machines from M^0 one by one
  - remove the schedule of the chosen machine and calculate a new schedule based on the earliest starting times and delays resulting from the other machines of M^0 and the job orders

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Technical realization rescheduling

For a chosen machine $l \in M^0 \setminus \{k\}$ do:

- remove the arcs corresponding to the selection on machine l from G'
- call new graph G''
- calculate values $r_{ij},\ q_{ij}$ in graph G''
- reschedule machine l according to the optimal schedule of the single machine head-body-tail problem

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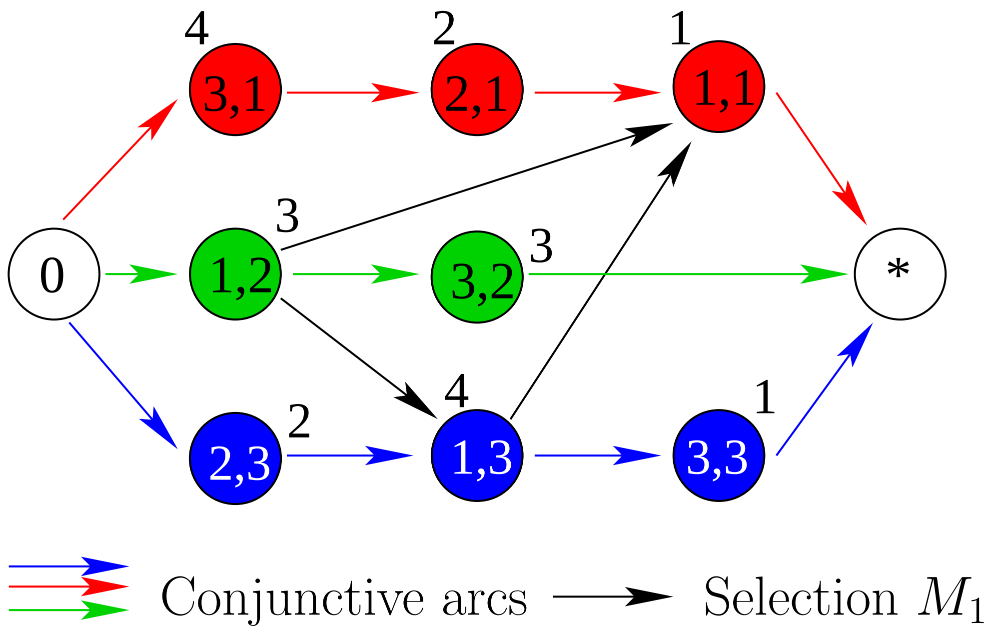
Technical realization rescheduling - Example

- $M^0 \setminus \{k\} = \{M_3\}$, thus $l = M_3$
- removing arcs corresponding to M_3 leads to graph G'':



$C_{max}(G'') = 8$;

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Technical realization rescheduling - Example

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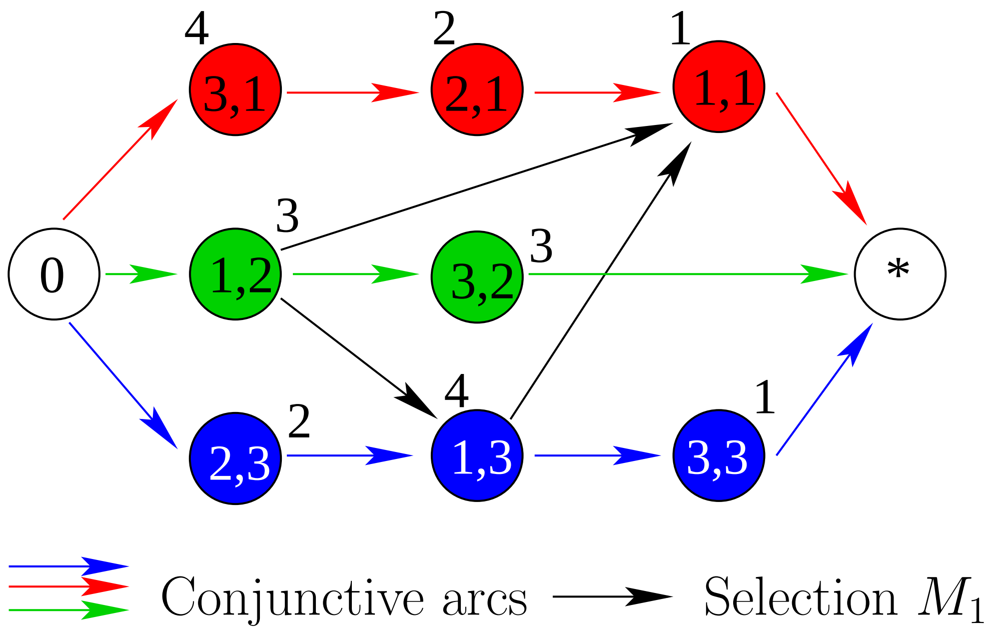
- removing arcs corresponding to M_3 leads to graph G'':



(i,j)	$(3,1)$	$(3,2)$	$(3,3)$
r_{ij}	0	3	7
q_{ij}	3	0	0
p_{ij}	4	3	1

$C_{max}(G'') = 8;$

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Technical realization rescheduling - Example

- $M^0 \setminus \{k\} = \{M_3\}$, thus $l = M_3$
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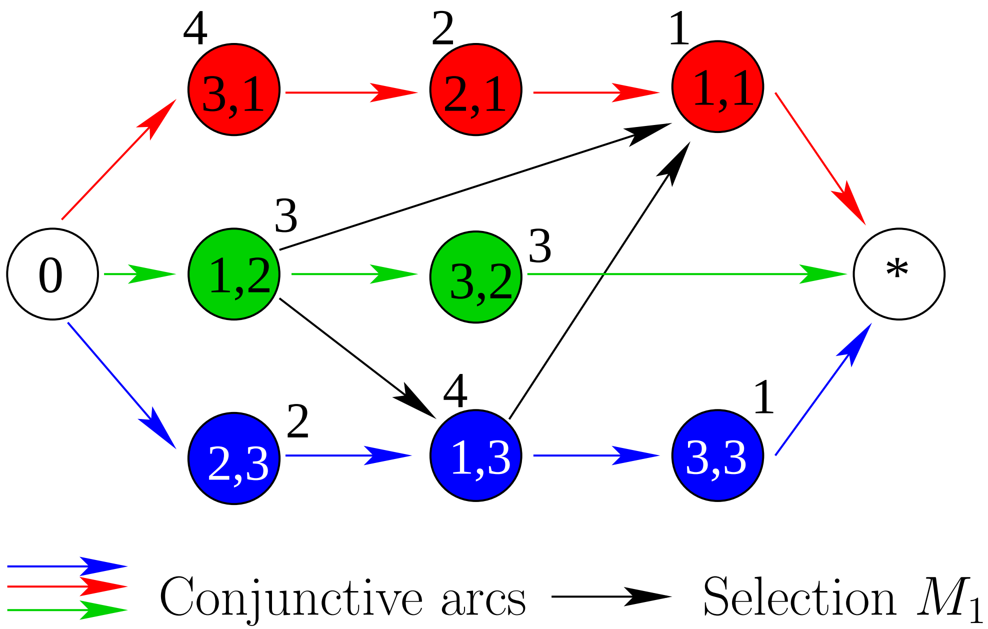

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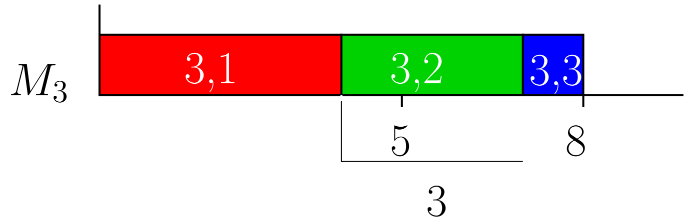

$C_{max}(G'') = 8;$

$f(M_3) = 0$

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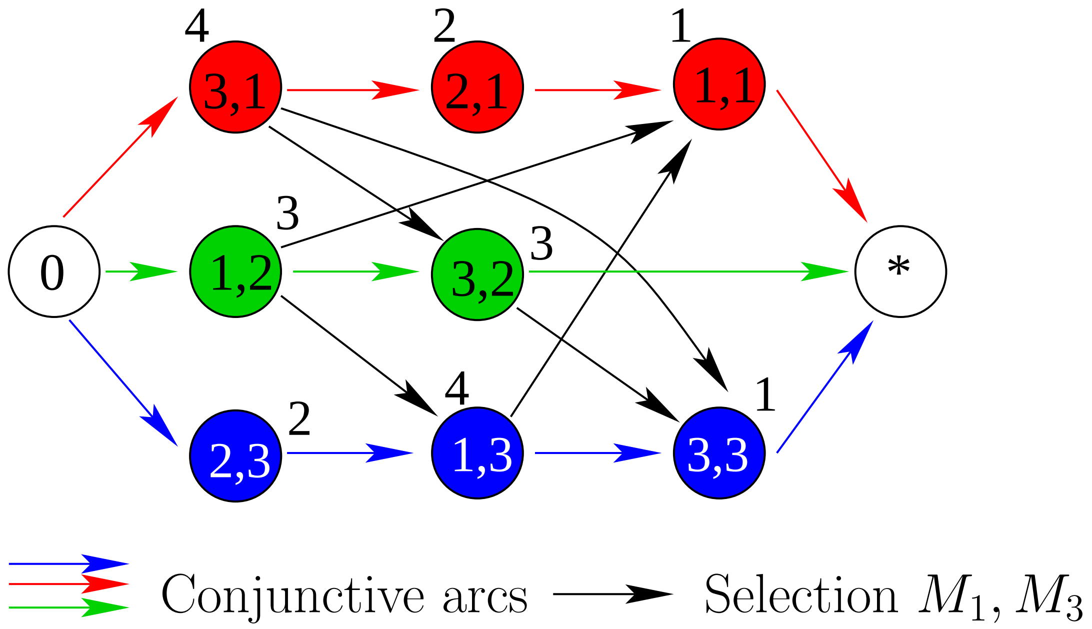
Technical realization rescheduling - Example (cont.)

- add $(3,1) \rightarrow (3,2) \rightarrow (3,3)$ to G''
- New graph:



- $C_{max}^{new}(M^0) = 8$

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Heuristic: summary

1. Initialization:
   (a) $M^0 := \emptyset$;
   (b) $G :=$ graph with all conjunctive arcs;
   (c) $C_{max}(M^0) :=$ length longest path in G:
2. Analyze unscheduled machines:

   FOR ALL $i \in M \setminus M^0$ DO

   FOR ALL operation (i, j) DO

   (a) $r_{ij} :=$ length longest path from 0 to (i, j) in G;

   (b) $q_{ij} :=$ length longest path from (i, j) to $*$ in G;

   solve single machine head body tail problem $\rightarrow f(i)$

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Heuristic: summary (cont.)

3. Bottleneck selection:
   (a) determine k such that $f(k) = \max_{i \in M \setminus M^0} f(i)$;
   (b) schedule machine k according to the optimal solution in Step 2;
   (c) add corresponding disjunctive arcs to G;
   (d) $M^0 := M^0 \cup \{k\}$;

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Heuristic: summary (cont.)

4. Resequencing of machines:

   FOR ALL $i \in M^0 \setminus \{k\}$ DO

   (a) delete disjunctive arcs corresponding to machine k from G;

   (b) FOR ALL operation (i, j) DO

      i. r_{ij} := length longest path from 0 to (i, j) in G;

      ii. q_{ij} := length longest path from (i, j) to $*$ in G;

   (c) solve single machine head body tail problem $\rightarrow f(i)$

   (d) insert corresponding disjunctive arcs to G;

5. Stopping condition

   IF $M^0 = M$ THEN Stop ELSE go to Step 2;

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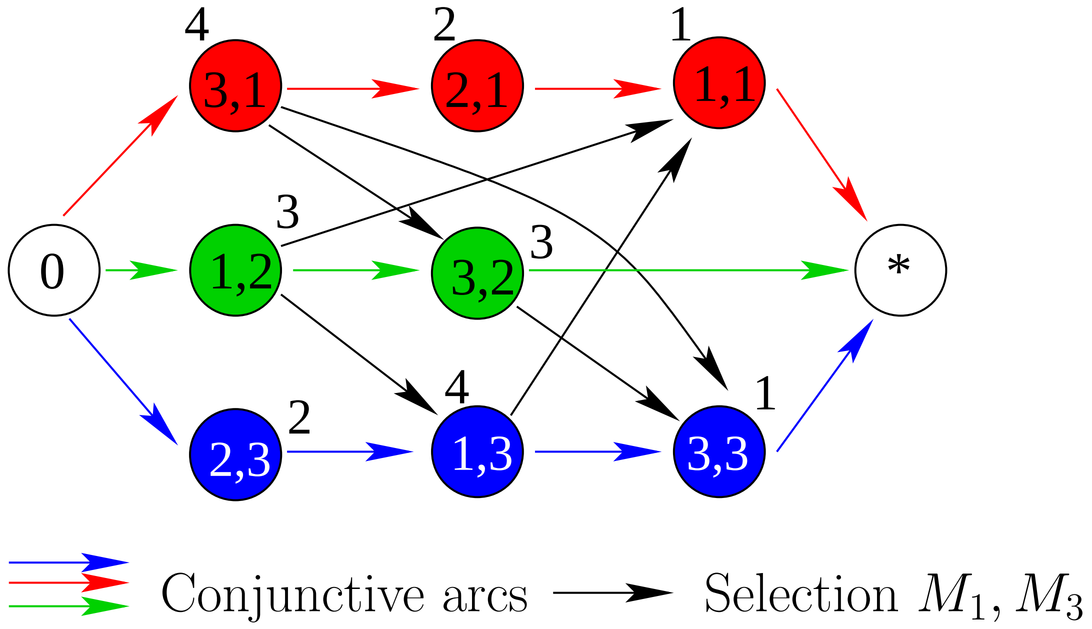
SBH - Example (cont.)

- $M^0 = \{M_1, M_3\}$; thus M_2 is bottleneck
- graph G:



- $C_{max}(G) = 8$;

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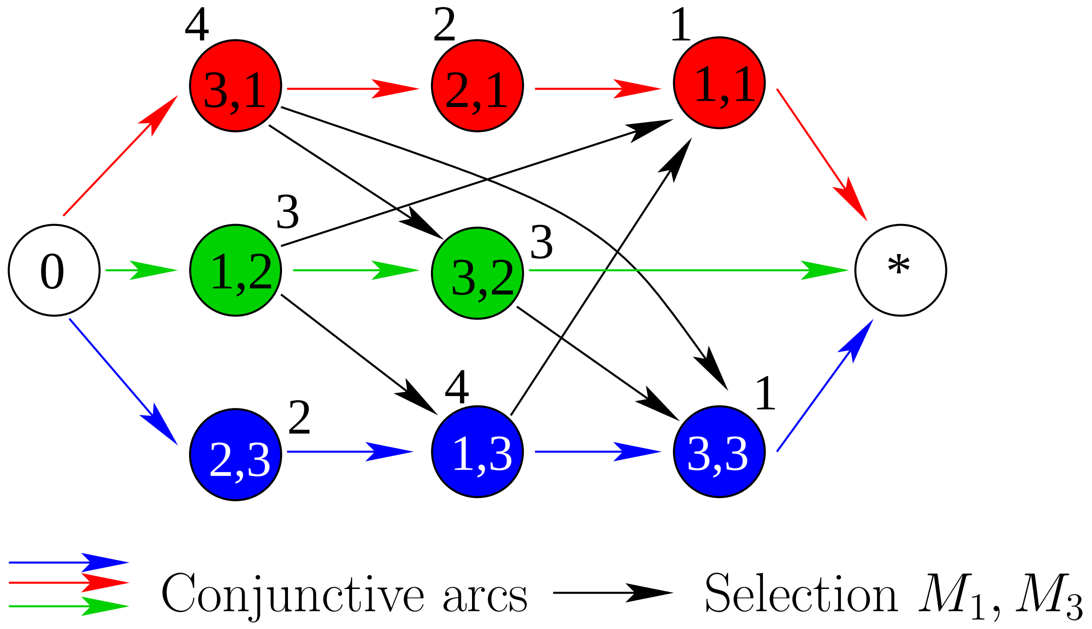
SBH - Example (cont.)

- $M^0 = \{M_1, M_3\}$; thus M_2 is bottleneck
- graph G:



(i,j)	$(2,1)$	$(2,3)$
r_{ij}	4	0
q_{ij}	1	5
p_{ij}	2	2

- $C_{max}(G) = 8$;

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SBH - Example (cont.)

- $M^0 = \{M_1, M_3\}$; thus M_2 is bottleneck
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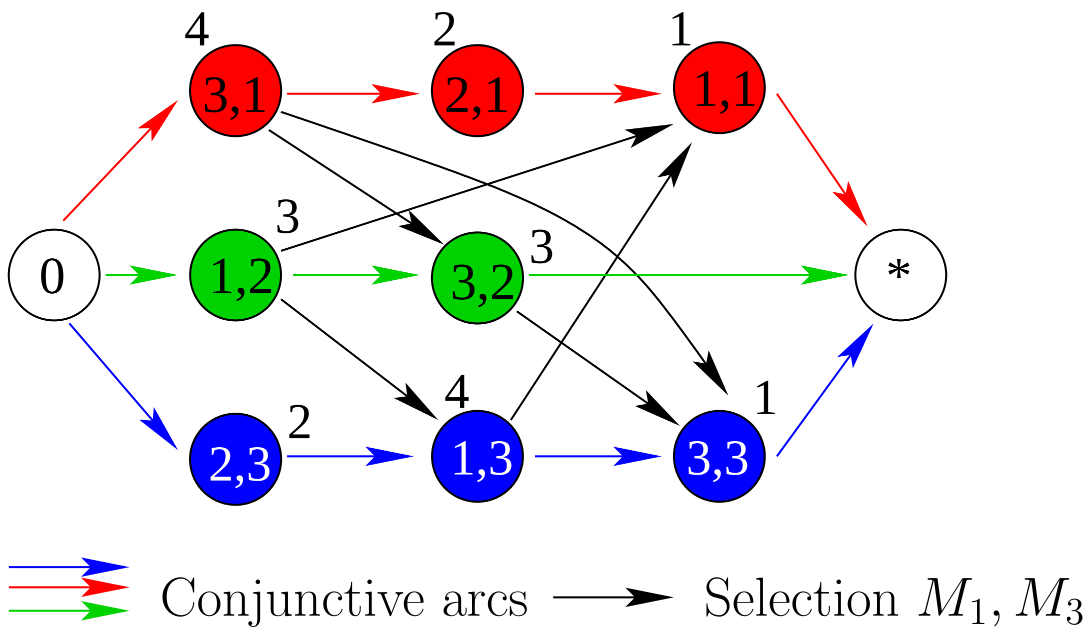

(i, j)	$(2, 1)$	$(2, 3)$
r_{ij}	4	0
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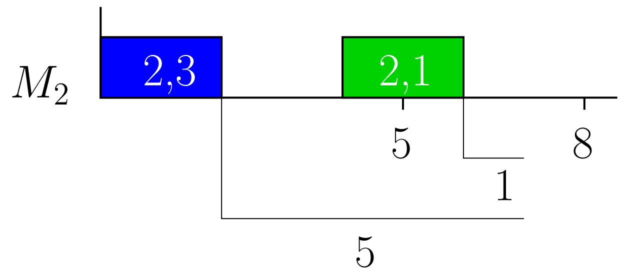
p_{ij}	2	2



$f(M_2) = 7$

- $C_{max}(G) = 8$;

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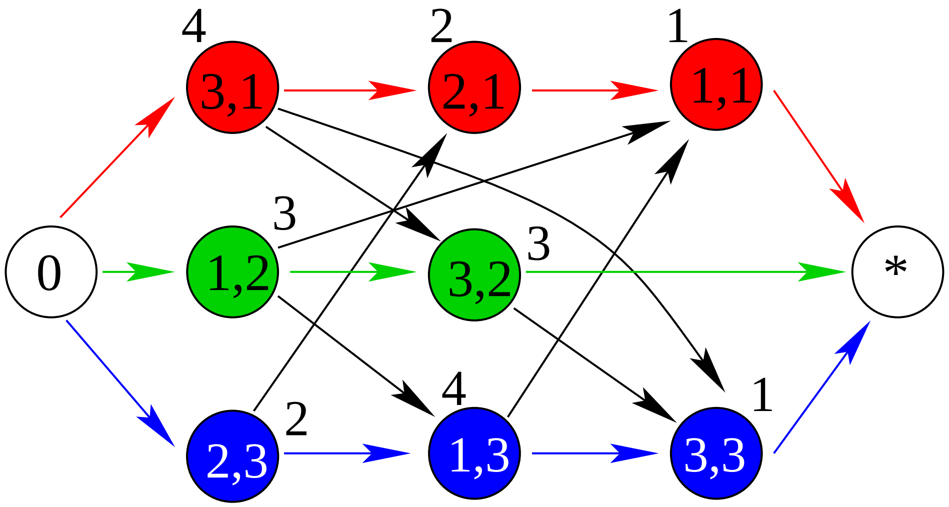
SBH - Example (cont.)

- Situation after Step 3: $M^0 = \{M_1, M_2, M_3\}$, $C_{max}(M^0) = 8$
- Graph G:



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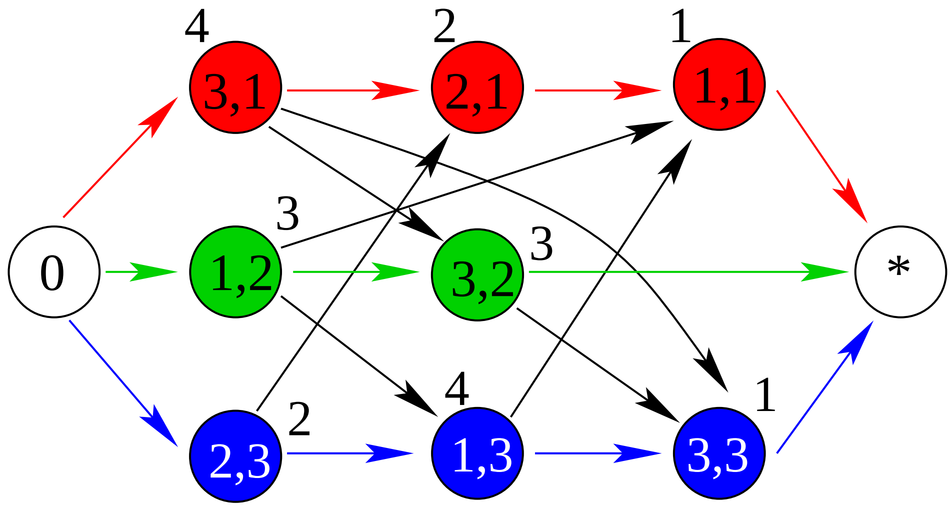
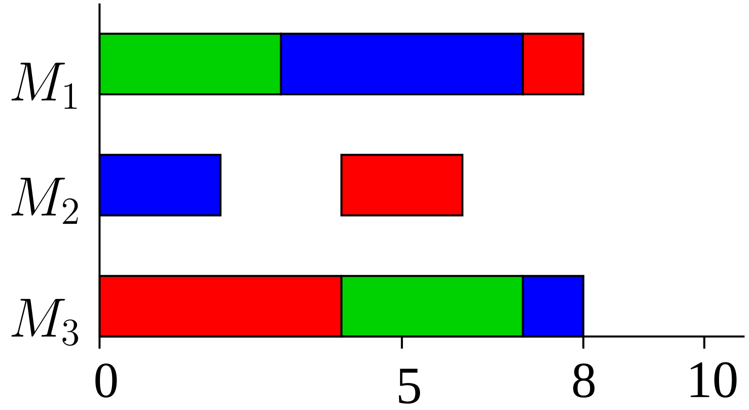
SBH - Example (cont.)

- Situation after Step 3: $M^0 = \{M_1, M_2, M_3\}$, $C_{max}(M^0) = 8$
- Graph G:



- Corresponding Schedule:



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An Important Subproblem

- within the SBH the one-machine head-body-tail problem occurs frequently:
- this problem was also used within branch and bound to calculate lower bounds
- the problem is NP-hard (see Lecture 3)
- there are efficient branch and bound methods for smaller instances (see also Lecture 3)
- the actual one-machine problem is a bit more complicated than stated in Lecture 3 (see following example)

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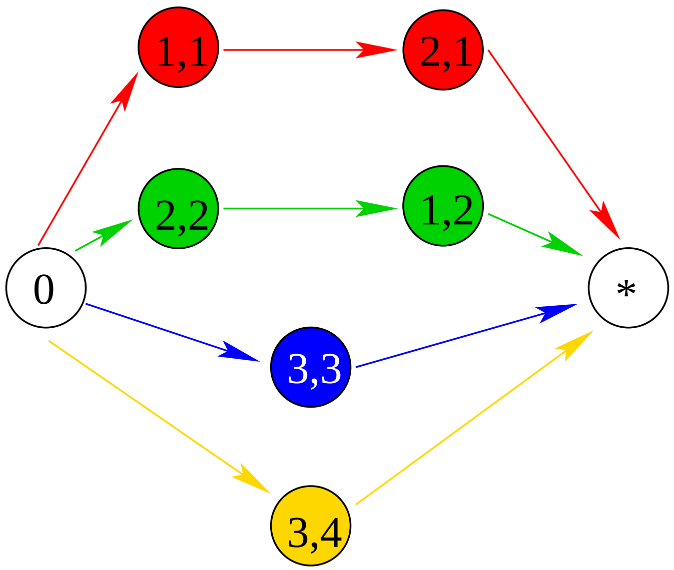
Example Delayed Precedences

Jobs:

- (red) $(1,1) \to (2,1)$
- (green) $(2,2) \to (1,2)$
- (blue) $(3,3)$
- (yellow) $(3,4)$

Processing Times: $p_{11} = 1,\ p_{21} = 1$
$p_{22} = 1,\ p_{12} = 1$
$p_{33} = 4$
$p_{34} = 4$

Initial graph G:



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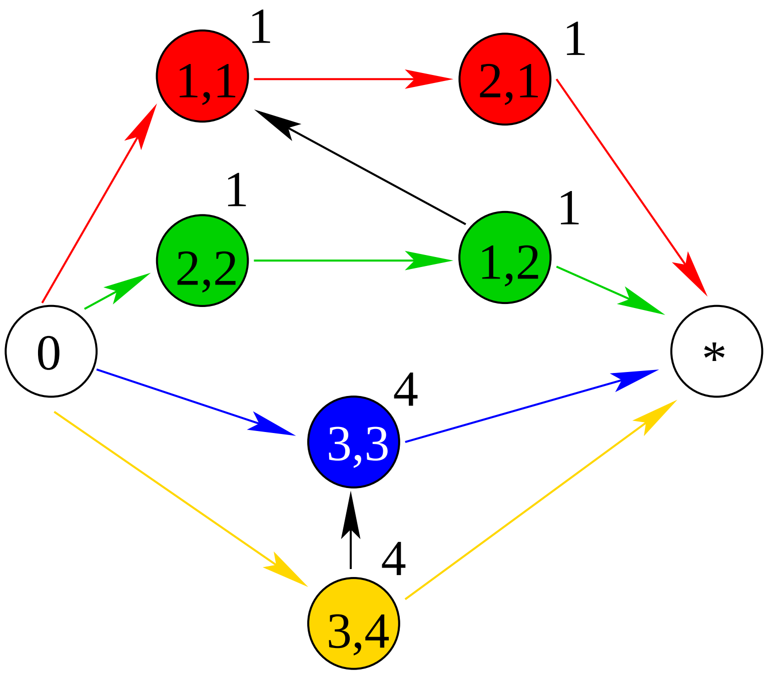
Example Delayed Precedences (cont.)

- after 2 iterations SBH we get: $M^0 = \{M_3, M_1\}$; $(3,4) \rightarrow (3,3)$ and $(1,2) \rightarrow (1,1)$
- Resulting graph G: $(C_{max}(M^0) = 8)$



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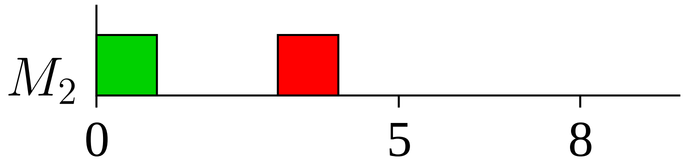
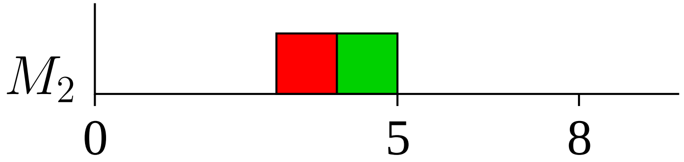
Example Delayed Precedences (cont.)

- 3. iteration: only M_2 unscheduled

(i, j)	p_{ij}	r_{ij}	q_{ij}
$(2, 1)$	1	3	0
$(2, 2)$	1	0	3

- Possible schedules for M_2:





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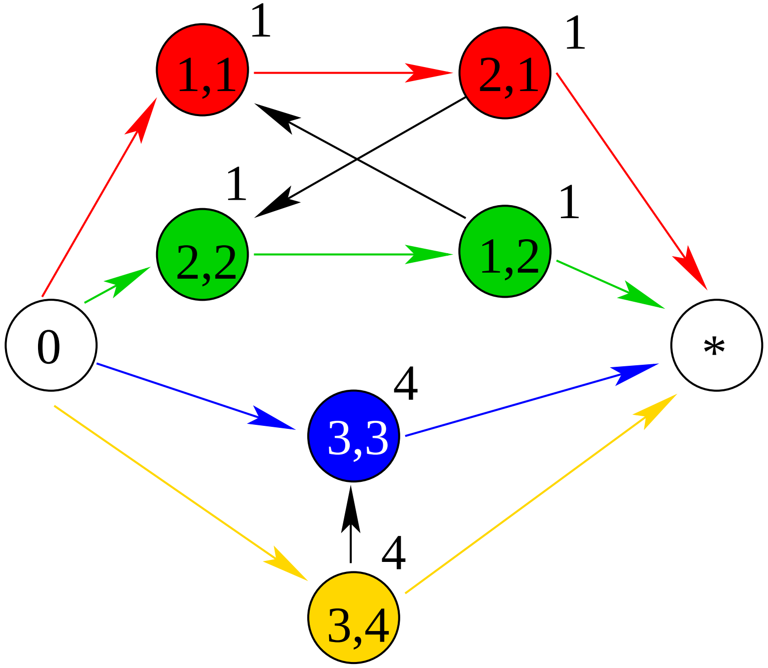
Example Delayed Precedences (cont.)

- Both schedules are feasible and might be added to the current solution
- But: second schedule leads to



which contains a cycle

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Delayed Precedences

- The example shows:
  - not all solutions of the one-machine problem fit to the given selections for machines from M^0
  - the given selections for machines from M^0 may induce precedences for machines from $M \setminus M^0$
- Example:
  - scheduling operation $(1, 2)$ before $(1, 1)$ on machine M_1 induces a delayed precedence constraint between $(2, 2)$ and $(2, 1)$ of length 3
  - $\longrightarrow$ operation $(2, 1)$ has to start at least 3 time units after $(2, 2)$
  - this time is needed to process operations $(2, 2), (1, 2)$, and $(1, 1)$

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Rescheduling Machines

- after adding a new machine to M^0, it may be worth to put more effort in rescheduling the machines:
  - do the rescheduling in some specific order (e.g. based on their 'head-body-tail' values)
  - repeat the rescheduling process until no improvement is found
  - after rescheduling one machine, make a choice which machine to reschedule next (allowing that certain machines are rescheduled more often)
  - ...
- practical test have shown that these extra effort often pays off