PLANNING AND SCHEDULING

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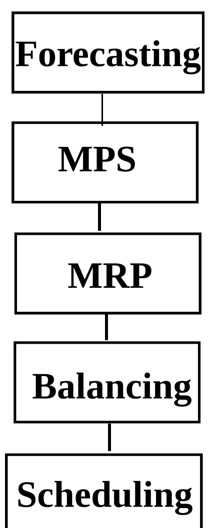
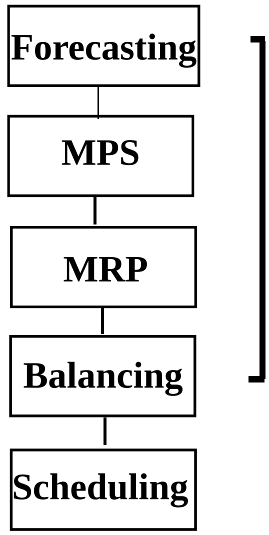
Planning Hierarchy



- Forecasting
- Master Production Planning (Scheduling)
- Material Requirements Planning (MRP) ⇨ **ERP**
- Capacity Balancing
- Production Scheduling

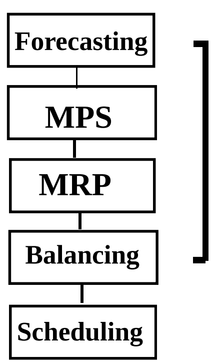


MRP II (Manufacturing Resource Planning II)

ERP = MRP II + ...

History of ERP

- 1970's MRP Material Requirements Planning
- 1980's MRPII Manufacturing Resource Planning
- 1990's ERP Enterprise Resource Planning (e.g., SAP system)



MRP II ⇒ **ERP**

Addition of CRM

Master Production Schedule
specifies
Sequence and Quantity of Products (C)

MPS
MRP
Balancing
Scheduling

EXAMPLE

Jan	Feb	March	Month
200 C1	195 C4	385 C1	
150 C7	150 C7	160 C6	
180 C14	180 C12	670 C7	
	128 C17	230 C9	

ERP systems are used from

• Automotive industry

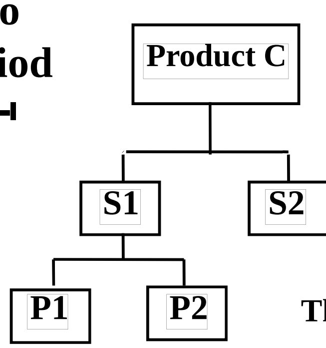
to

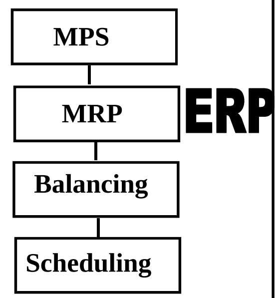
• Pharmaceutical industry

MRP ERP

Planning horizon:

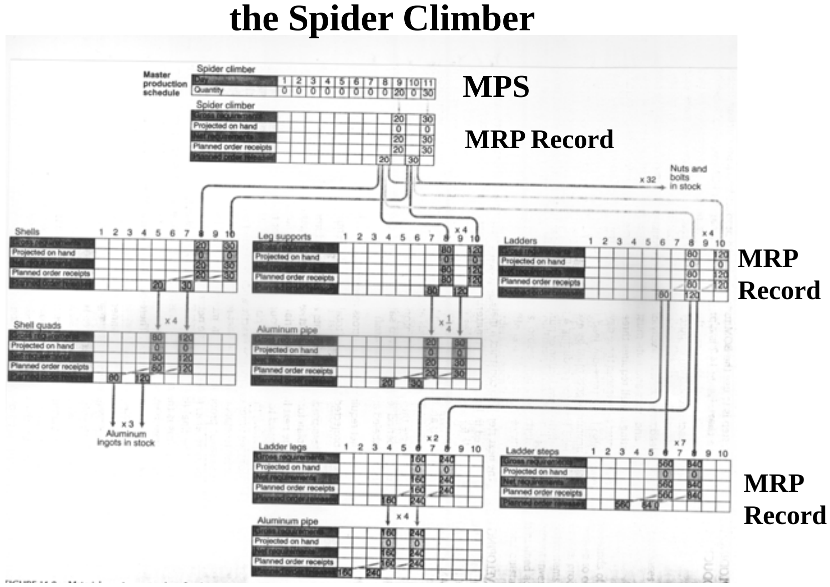
1 month to
3 day period





The BOM of Product C

EXAMPLE: Material Requirements Records for the Spider Climber



Merged Material Requirements for Aluminum Pipe

Aluminum pipe for leg supports	1	2	3	4	5	6	7	8	9	10
Gross requirements							20		30	
Projected on hand							0		0	
Net requirements							20		30	
Planned order receipts							20		30	
Planned order releases				20		30				

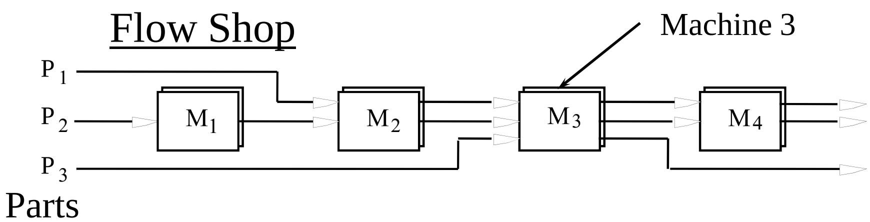
Aluminum pipe for ladder legs	1	2	3	4	5	6	7	8	9	10
Gross requirements				160		240				
Projected on hand				0		0				
Net requirements				160		240				
Planned order receipts				160		240				
Planned order releases	160		240							

Total aluminum pipe	1	2	3	4	5	6	7	8	9	10
Gross requirements				160		240	20		30	
Projected on hand				0		0	0		0	
Net requirements				160		240	20		30	
Planned order receipts				160		240	20		30	
Planned order releases	160		240	20		30				

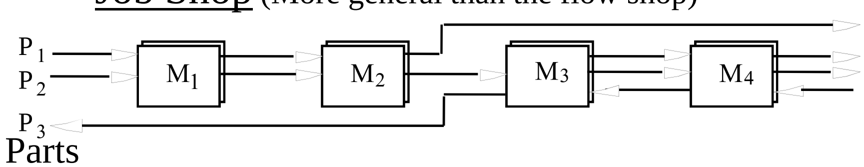
MRP Record

Basic Scheduling Models

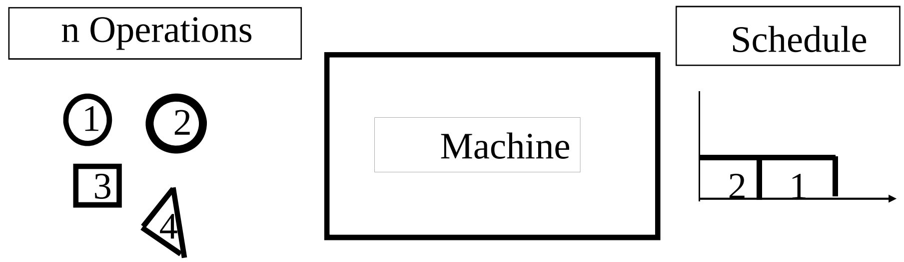
Flow Shop



Job Shop (More general than the flow shop)



Single Machine Scheduling



Jobs
Tasks
Computer programs

Lathe
Computer
Bank teller

Scheduling n Operations on a Single Machine

Case 1: No constraints are imposed

Theorem 1 (SPT Rule)

For a one-machine scheduling problem, the mean flow time is minimized by the following sequence:

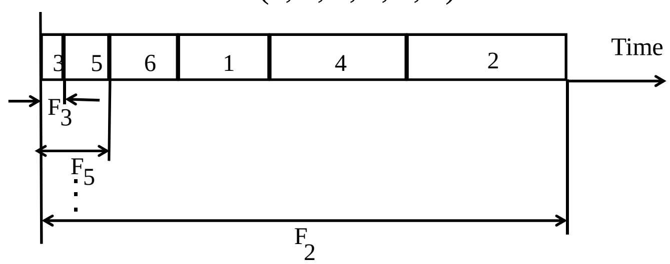
$$t\,(1) \leq t\,(2) \leq t\,(3)\ \ldots \leq t\,(i)\ \ldots \leq t\,(n)$$

where t (i) is the processing time of the operation that is processed i^{th}

Example

Operation number	1	2	3	4	5	6
Processing time	4	7	1	6	2	3

SOLUTION: Gantt Chart of Single Machine
Schedule (3, 5, 6, 1, 4, 2)



Flow time F3 = 1, F5 = 3, F6 = 6, F1 = 10, F4 = 16, F2 = 23

The mean flow time F = 9.83 is minimum

?

What does the minimization of the mean flow time imply?

Completing tasks with the minimum average flow time

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Case 2: Due dates are imposed

Theorem 2 (EDD Rule)

For the one - machine scheduling problem with due dates, the maximum lateness is minimized by sequencing such that:

$$d(1) \le d(2) \le d(3) \le \ldots\ d(i) \le \ldots\ d(n)$$

where d(i) is the due date of operation that is processed ith

Example

Operation number	1	2	3	4	5	6
Processing time	1	1	2	5	1	3
Due date	6	3	8	14	9	3

Operation number	1	2	3	4	5	6
Processing time	1	1	2	5	1	3
Due date	6	3	8	14	9	3

Operation Number	Due Date	Completion Time	Lateness	Tardiness
6	3	3	0	0
2	3	4	1	1
1	6	5	-1	0
3	8	7	-1	0
5	9	8	-1	0
4	14	13	-1	0

The optimal EDD schedule is (6, 2, 1, 3, 5, 4)

Operation 2 is late ($L_2 = 1$)

?

What does the minimization of the maximum lateness imply?

Elimination of long delays
&
More even distribution of delays (balancing delays)

Classroom Exercise

• Job No.	1	2	3
• Proc Time	7	4	12
• Due time	16	10	9

- Find SPT Schedule
- Find EDD schedule

SPT {2, 1, 3}

EDD {3, 2, 1}

SCHEDULING MODELS

- Two-Machine Flowshop
- Two-Machine Job Shop
- Extensions

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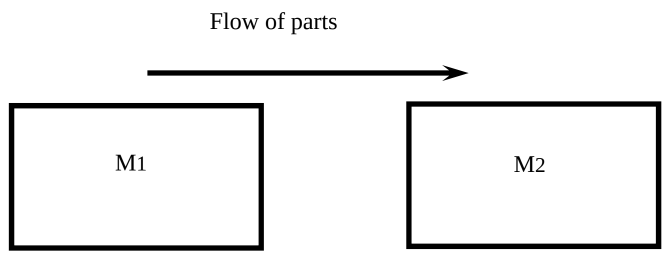
Two-Machine Flowshop



Modified Johnson's Algorithm

Minimization of Max Flow (Min F_{Max})

?

What does Minimization of the Max Flow (Min FMax) imply?

Neutralizing outliers

Two-Machine Flowshop Model

Modified Johnson's Algorithm

Step 1. *Set k = 1, l = n.*
Step 2. *For each part, store the shortest processing time and the corresponding machine number.*
Step 3. *Sort the resulting list, including the triplets "part number/time/machine number" in increasing value of processing time.*
Step 4. *For each entry in the sorted list:*
IF machine number is 1, then
(i) set the corresponding part number in position k,
(ii) set k = k + 1.
ELSE
(i) set the corresponding part number in position l,
(ii) set l = l - 1.
END-IF.
Step 5. *Stop if the entire list of parts has been exhausted.*

Example: Two-Machine Flowshop Model

Schedule 7 parts

Two operations per part, each performed on a different machine

Part Number i	**Processing Time** t_{ij} **of Part** i **on Machine** j $j = 1$	$j = 2$
1	6	3
2	2	9
3	4	3
4	1	8
5	7	1
6	4	5
7	7	6

Min	**M**
3	2
2	1

For each part calculate Min {ti1, ti2}

Min processing time calculated

Part Number	min $\{t_{i1}, t_{i2}\}$	Machine Number
1	3	2
2	2	1
3	3	2
4	1	1
5	1	2
6	4	1
7	6	2

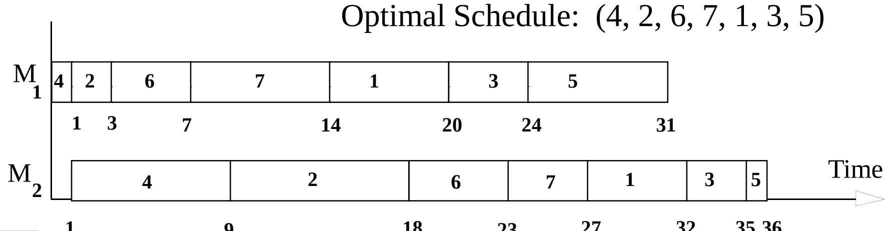
Triplets ordered on the processing time

(4, 1, 1)
(5, 1, 2)
(2, 2, 1)
(3, 3, 2)
(1, 3, 2)
(6, 4, 1)
(7, 6, 2)

(4, 1, 1)
(5, 1, 2)
(2, 2, 1)
(3, 3, 2)
(1, 3, 2)
(6, 4, 1)
(7, 6, 2)

Machine 1 → Machine 2 ←

Optimal Schedule: (4, 2, 6, 7, 1, 3, 5)



?

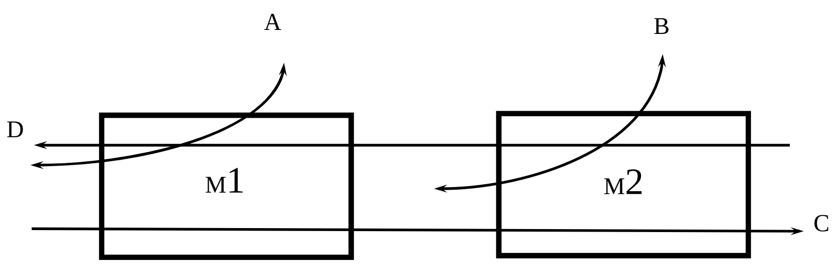
Think of incorporating

in Johnson's Algorithm

✓ Due dates
✓ Precedence constraints

Two-Machine Job Shop

Use of Johnson's algorithm by dividing parts into <u>four types</u>



Two-Machine Job Shop

Type A: parts to be processed only on machine M1.

Type B: parts to be processed only on machine M2.

Type C: parts to be processed on both machines in the order M1, M2.

Type D: parts to be processed on both machines in the order M2, M1.

Algorithm

Step 1. Schedule the parts of type A in any order to obtain the sequence SA.

Step 2. Schedule the parts of type B in any order to obtain the sequence SB.

Step 3. Scheduling the parts of type C according to Johnson's algorithm produces the sequence SC.

Step 4. Scheduling the parts of type D according to Johnson's algorithm produces the sequence SD (Note that M2 is the first machine, whereas M1 is the second one).

Step 5. Construct an optimal schedule as follows:

The Optimal Schedule

M1	(SC, SA, SD)
M2	(SD, SB, SC)

Example: Two-Machine Job Shop

Processing Time

Parts

Parts	First machine		Second machine	
1	M_1	7	M_2	1
2	M_1	6	M_2	5
3	M_1	9	M_2	7
4	M_1	4	M_2	6
5	M_2	6	M_1	6
6	M_2	5	M_1	5
7	M_1	4	-	-
8	M_1	5	-	-
9	M_2	1	-	-
10	M_2	5	-	-

Parts

Parts	First machine		Second machine	
1	M_1	7	M_2	1
2	M_1	6	M_2	5
3	M_1	9	M_2	7
4	M_1	4	M_2	6
5	M_2	6	M_1	6
6	M_2	5	M_1	5
7	M_1	4	-	-
8	M_1	5	-	-
9	M_2	1	-	-
10	M_2	5	-	-

Type A: parts to be processed only on machine M1.

Type B: parts to be processed only on machine M2.

Type A parts: Parts 7 and 8 are to be processed on machine M1 alone. An arbitrary order SA = (7, 8) is selected.

Type B parts: Parts 9 and 10 require machine M2 alone. Select an arbitrary order SB = (9, 10).

Parts

	First machine		Second machine	
1	M_1	7	M_2	1
2	M_1	6	M_2	5
3	M_1	9	M_2	7
4	M_1	1	M_2	6
5	M_2	6	M_1	6
6	M_2	5	M_1	5
7	M_1	4	-	-
8	M_1	5	-	-
9	M_2	1	-	-
10	M_2	5	-	-

Type C parts: Parts 1, 2, 3, and 4 require machine M1 first and then machine M2.

Type C Parts

Part Number	First Machine	Second Machine
1	7	1
2	6	5
3	9	7
4	1	6

Min {ti1, ti2}

1	1	2
2	5	2
3	7	2
4	1	1

4	1	1
1	1	2
2	5	2
3	7	2

$S_C = (4, 3, 2, 1)$

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Parts

	First machine		Second machine	
1	M_1	7	M_2	1
2	M_1	6	M_2	5
3	M_1	9	M_2	7
4	M_1	4	M_2	6
5	M_2	6	M_1	6
6	M_2	5	M_1	5
7	M_1	4	-	-
8	M_1	5	-	-
9	M_2	1	-	-
10	M_2	5	-	-

Type D parts: Parts 5 and 6 require machine M2 first and then machine M1.

Type D Parts

Part Number	First Machine	Second Machine
5	6	6
6	5	5

Min {ti1, ti2}

5	6	1st(M2)
6	5	1st(M2)

6	5	1st (M2)
5	6	1st (M2)

$S_D = (5, 6)$

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Optimal Schedule

M1: (S_C, S_A, S_D)
M2: (S_D, S_B, S_C)

Partial Schedules

S_A = (7, 8) S_B = (9, 10)
S_C = (4, 3, 2, 1) S_D = (5, 6)

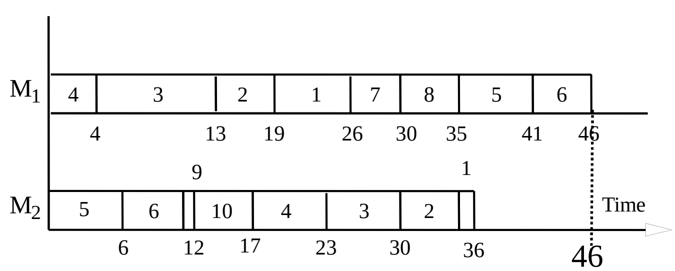
Optimal Schedule

M1: (4, 3, 2, 1, 7, 8, 5, 6)
M2: (5, 6, 9, 10,4, 3, 2, 1)

Optimal Schedule

M1: (4, 3, 2, 1, 7, 8, 5, 6)
M2: (5, 6, 9,10,4, 3, 2, 1)

Gantt Chart of the Optimal Schedule



Min F_{max} = 46 for the optimal schedule

?

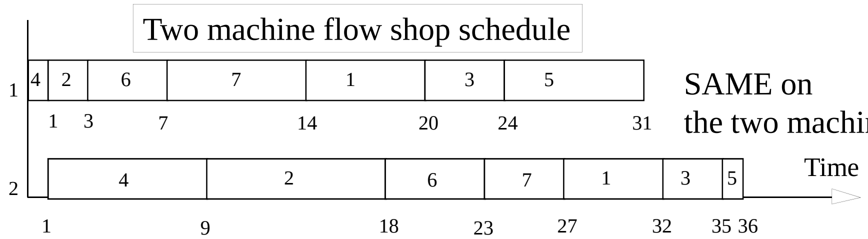
What is the main difference between

✓ Two machine flow shop schedule
  and
✓ Two machine job shop schedule

both solved with Johnson's algorithm?

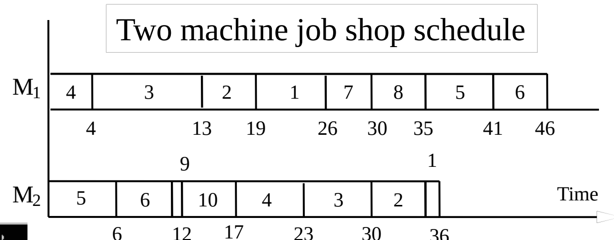
Answer: The Sequence of Operations

Two machine flow shop schedule



SAME on
the two machines

Two machine job shop schedule



DIFFERENT
on each machine

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Special Case of Three-Machine Flow Shop Model

Either $\min_{i=1}^{n} \{t_{i1}\} \geq \max_{i=1}^{n} \{t_{i2}\}$

or $\min_{i=1}^{n} \{t_{i3}\} \geq \max_{i=1}^{n} \{t_{i2}\}$

$$\boxed{\begin{aligned} a_i &= t_{i1} + t_{i2} \\ b_i &= t_{i2} + t_{i3} \end{aligned}}$$

Example: Special Case of Three-Machine Flow Shop Model

Scheduling Data

Actual Processing Times

Part	t_{i1} M_1	t_{i2} M_2	t_{i3} M_3
1	4	1	2
2	6	2	10
3	3	1	2
4	5	3	6
5	7	2	6
6	4	1	1

Part	Actual Processing Times t_{i1} M_1	t_{i2} M_2	t_{i3} M_3	→	Constructed Processing Times a_i First Machine	b_i Second Machine
1	4	1	2		5	3
2	6	2	10		8	12
3	3	1	2		4	3
4	5	3	6		8	9
5	7	2	6		9	8
6	4	1	1		5	2

$$\min_{i=1}^{6}\{t_{i1}\} = 3; \quad \max_{i=1}^{6}\{t_{i2}\} = 3; \quad \text{and} \quad \min_{i=1}^{6}\{t_{i3}\} = 1$$

The first condition is met

$$\boxed{\min_{i=1}^{6}\{t_{i1}\} = 3 \geq 3 = \max_{i=1}^{6}\{t_{i2}\}}$$

Part Number	First Machine	Second Machine
1	5	3
2	8	12
3	4	3
4	8	9
5	9	8
6	5	2

→

Min {ti1, ti2}

1	3	2
2	8	1
3	3	2
4	8	1
5	8	2
6	2	2

6	2	2
3	3	2
1	3	2
2	8	1
4	8	1
5	8	2

→ (2, 4, 5, 1, 3, 6)

Gantt Chart of the Optimal Solution

Optimal Solution (2, 4, 5, 1, 3, 6)