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Some Common Performance Measures in Scheduling Problems: Review Article

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Abstract: In this Study, we discussed 29 different scheduling criteria. We developed mathematical expressions for all the criteria considered. Each of the criteria was expressed as a function of either the completion time of job or the given parameters. This will assist researchers to easily compute the value of any of the scheduling criteria considered in this study.

Keywords: Machine, mathematical expressions, performance measures, scheduling, solution methods, total completion time

INTRODUCTION

Scheduling objectives (also called performance measures) are criteria by which the performance of any solution method can be measured. It is not easy to state scheduling objectives (French, 1982; Albert and Rabelo, 1998). This is because they are numerous, complex and often conflicting. Scheduling objectives could be completion time based, due date based or inventory based (Kanet and Sridharan, 2000; Dauzere-Perez and Sevaux, 2003). The objectives often vary from firm to firm.

Literature on the extensive study of scheduling objectives is sparse. Conway et al., (1960) cited four criteria why Gere (1962) listed about seven scheduling criteria. Perhaps only Beenhakker (1963) has made an attempt by coming out with an extensive list of about 27 distinct scheduling goals (objectives) (Mellor, 1966). However, Conway et al. (1960), Gere (1962), Beenhakker (1963) failed to formulate the mathematical expressions that can be used to compute the considered scheduling objectives. It is important for researchers working on scheduling problems to know how each of the scheduling objectives can be computed. This study addressed this major gap by formulating from first principle mathematical expressions for about 29 distinct scheduling objectives.

Scheduling performance measures: In formulating the mathematical expressions for the scheduling objectives considered in this paper, the following notations are used:

C_i = completion time of job i
F_i = flow time of job i
L_i = lateness of job i
w_i = relative weight of job i
r_i = release date of job i
p_i = processing time of job i
d_i = due date of job i
T_i = Tardiness of job i
E_i = Earliness of job i
MIN-SUM = Minimize the sum of a quantity
MIN-MAX = Minimize the maximum of a quantity
MAX-SUM = Maximize the sum of a quantity
MAX-MAX = Maximize the maximum of a quantity

Performance measures based on completion time: The completion time of job J_i is the time at which the processing of the job J_i finishes. For a multi operation job, it is the time the last operation of the job J_i finished. The scheduling criteria based on the completion time of jobs are as follows:

Total completion time (C_{tot}) = $\sum_{i=1}^{n} C_i$

The total completion time (C_{tot}) is the sum of all the completion times of the jobs. A common problem is to minimize the total completion time. This leads to what is referred to as MIN-SUM problem.

Total weighted completion time (wC_{tot}) = $\sum_{i=1}^{n} W_i C_i$

This is the sum of all the completion times multiplied by the relative weights of the jobs. A common problem is in minimizing the total weighted completion time. This problem allows one to find an indication to the total holding or inventory caused by the schedule (Hochbaum, 1999). This also leads to MIN-SUM problem.

Average completion time (C_{avg}) = $\frac{1}{n}\sum_{i=1}^{n} C_i$

The average completion time gives the average time it takes to complete each job. A common problem is to minimize the average completion time. This leads to MIN-SUM problem.

Average weighted completion time

$$(wC_{avg}) = \frac{1}{n}\sum_{i=1}^{n} W_i C_i$$

This the total weighted completion time divided by the number of jobs. Since the number of jobs in any particular problem instance is constant, the total weighted completion time criterion is equivalent to the total weighted completion time criterion. A common problem involves minimization of the total weighted completion time. This leads to a MIN-SUM problem.

Maximum completion time
$(C_{max}) = \max (C_1, C_2, \ldots, C_n)$

The maximum completion time (also called makespan) is the completion time of the last job. A common problem of interest is to minimize C_{max}, or to minimize the completion time of the last job to leave the system. This criterion is usually used to measure the level of utilization of the machine. This leads to MIN-MAX problem.

Performance measures based on flow time: ($F_i = C_i - r_i$)

The flow time of job J_i is the time that job J_i spends in the workshop. It is the time interval between the time the job is released to the shop and the time the processing of the job is completed. Scheduling criteria based on the flow time of jobs are listed below.

Total flow time

$$(F_{tot}) = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} (C_i - r_i) = \sum_{i=1}^{n} C_i - \sum_{i=1}^{n} r_i$$

The total flow time (F_{tot}) is the sum of all the flow times of the jobs. A common problem is to minimize the total flow time. This leads to a MIN-SUM problem.

Total weighted flow time

$$(WF_{tot}) = \sum_{i=1}^{n} w_i F_i = \sum_{i=1}^{n} w_i C_i - \sum_{i=1}^{n} w_i r_i$$

This is the sum of all the flow times multiplied by the relative weights of the jobs. A common problem is in minimizing the total weighted flow time. This leads to a MIN-SUM problem.

Average flow time

$$(F_{avg}) = \frac{1}{n}\sum_{i=1}^{n} F_i = \frac{1}{n}\sum_{i=1}^{n} C_i - \frac{1}{n}\sum_{i=1}^{n} r_i$$

The average flow time gives the average time each job spends in the shop. A common problem is to minimize the average flow time. This also is a MIN-SUM problem. The average flow time criterion is equivalent to the total flow time criterion.

Average weighted flow time

$$(wF_{avg}) = \frac{1}{n}\sum_{i=1}^{n} w_i F_i = \frac{1}{n}\sum_{i=1}^{n} w_i C_i - \frac{1}{n}\sum_{i=1}^{n} w_i r_i$$

The usual problem considered is minimizing the average weighted flow time. Note that this is equivalent to minimizing the total weighted flow time. This is a MIN-SUM problem.

Maximum flow time $(F_{max}) = \max (F_1, F_2, \ldots, F_n)$
$F_{max} = \max\{(C_1 - r_1), (C_2 - r_2), \ldots, (C_n - r_n)\}$

The maximum flow time is the longest of the flow times of the jobs. A common problem of interest is to minimize F_{max}. This leads to MIN-MAX problem.

Performance measures based on lateness: ($L_i = C_i - d_i$)

This is the difference between the completion time and the due date (the date the job is expected to be delivered) of the job. Scheduling criteria based on the lateness of a job are listed below.

Total lateness

$$(L_{tot}) = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} (C_i - d_i) = \sum_{i=1}^{n} C_i - \sum_{i=1}^{n} d_i$$

The total lateness (L_{tot}) is the sum of all the lateness of the jobs. A common problem is to minimize the total lateness. This leads to a MIN-SUM problem.

Total weighted lateness

$$(wL_{tot}) = \sum_{i=1}^{n} w_i L_i = \sum_{i=1}^{n} w_i C_i - \sum_{i=1}^{n} w_i d_i$$

This is the sum of all the lateness multiplied by the relative weights of the jobs. A common problem is in minimizing the total weighted lateness. This leads to a MIN-SUM problem.

verage lateness

$$(L_{avg}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} C_i - \frac{1}{n}\sum_{i=1}^{n} d_i$$

The average lateness is total lateness divided by the number of jobs. Therefore, minimizing total lateness also minimizes the average lateness. A common problem is to minimize the average lateness. This is a MIN-SUM problem.

Average weighted lateness

$$(wL_{avg}) = \frac{1}{n}\sum_{i=1}^{n} w_i L_i = \frac{1}{n}\sum_{i=1}^{n} w_i C_i - \frac{1}{n}\sum_{i=1}^{n} w_i d_i$$

The average weighted lateness is total weighted lateness divided by the number of jobs. Also, minimizing total weighted lateness also minimizes the average weighted lateness. A common problem is to minimize the average weighted lateness. This is a MIN-SUM problem.

Maximum lateness $(L_{max}) = \max (L_1, L_2, \ldots, L_n)$
$L_{max} = \max\{(C_1 - d_1), (C_2 - d_2), \ldots, (C_n - d_n)\}$

The maximum lateness (L_{max}) is the longest of the lateness of the jobs. A common problem of interest is to minimize L_{max}. This leads to MIN-MAX problem.

Performance measures based on number of late/tardy jobs: ($L_i = C_i - d_i$)

A job is said to be late or tardy if it completes after its due date. Scheduling criteria based on number of tardy jobs are listed below.

$$\text{Let } U_i = \begin{cases} 1; C_i > d_i \\ 0; otherwise \end{cases}$$

$$\text{Number of tardy jobs (NT)} = \sum_{i=1}^{n} U_i$$

The number of tardy jobs measures the number of jobs (n) that are completed after their due dates. Typical problem of interest is to minimize the number of tardy jobs. Minimizing number of tardy jobs criterion is equivalent to maximizing number of early jobs criterion.

$$\text{Average number of tardy jobs } (NT_{avg}) = \frac{1}{n}\sum_{i=1}^{n} U_i$$

The average number of tardy jobs is the number of tardy jobs divided by the number of jobs (n). Typical problem of interest is to minimize the average number of tardy jobs. Minimizing the average number of tardy jobs criterion is equivalent to maximizing the average number of early jobs criterion.

Performance measures based on tardiness: {$T_i = \max(0, L_i)$ }

Tardiness is similar to lateness except that it carries only positive values. Whenever a job completes before its due dates, its lateness is negative while its tardiness is zero. Scheduling criteria based on tardiness are listed below.

$$\text{Total tardiness } (T_{tot}) = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \max\{0, (C_i - d_i)\}$$

The total tardiness (T_{tot}) is the sum of all the tardiness of the jobs. A common problem is to minimize the total tardiness. This leads to a MIN-SUM problem.

Total weighted tardiness

$$(wT_{tot}) = \sum_{i=1}^{n} w_i T_i = \sum_{i=1}^{n} w_i [\max\{0, (C_i - d_i)\}]$$

This is the sum of all the tardiness multiplied by the relative weights of the jobs. A common problem is in minimizing the total weighted tardiness. This leads to a MIN-SUM problem.

Average tardiness

$$(T_{avg}) = \frac{1}{n}\sum_{i=1}^{n} T_i = \frac{1}{n}\sum_{i=1}^{n} \max\{0, (C_i - d_i)\}$$

The average tardiness is total tardiness divided by the number of jobs. Therefore, minimizing total tardiness criterion also minimizes the average tardiness criterion. A common problem is to minimize the average tardiness. This is a MIN-SUM problem.

Average weighted tardiness

$$(wT_{avg}) = \frac{1}{n}\sum_{i=1}^{n} w_i T_i = \frac{1}{n}\sum_{i=1}^{n} w_i [\max\{0, (C_i - d_i)\}]$$

The average weighted tardiness is total weighted tardiness divided by the number of jobs. Minimizing total weighted tardiness also minimizes the average weighted tardiness. A common problem is to minimize the average weighted tardiness. This is a MIN-SUM problem.

Maximum tardiness(T_{max}) = max (T_1 , T_2 , …, T_n)
T_{max} = max {0, (C_1 - d_1), (C_2 - d_2), …, (C_n - d_n)}

The maximum tardiness (T_{max}) is the longest of the tardiness of the jobs. A common problem of interest is to minimize T_{max}. This leads to MIN-MAX problem.

Performance measures based on earliness: (E_i ={ d_i -C_i })

Earliness is the opposite of lateness; hence, whenever lateness is negative earliness is positive and whenever lateness is positive earliness is negative.The following are the scheduling criteria that are based on earliness.

Total earliness

$$(E_{tot}) = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} (d_i - C_i) = \sum_{i=1}^{n} d_i - \sum_{i=1}^{n} C_i$$

The total earliness (E_{tot}) is the sum of all the earliness of the jobs. A common problem is to maximize the total earliness. This leads to a MAX-SUM problem.

Total weighted earliness

$$(wE_{tot}) = \sum_{i=1}^{n} w_i E_i = \sum_{i=1}^{n} w_i d_i - \sum_{i=1}^{n} w_i C_i$$

This is the sum of all the earliness multiplied by the relative weights of the jobs. A common problem is in maximizing the total weighted earliness. This leads to a MAX-SUM problem.

Average earliness

$$(E_{avg}) = \frac{1}{n}\sum_{i=1}^{n} E_i = \frac{1}{n}\sum_{i=1}^{n} d_i - \frac{1}{n}\sum_{i=1}^{n} C_i$$

The average earliness is total earliness divided by the number of jobs. A common problem is to maximize the average earliness. Therefore, maximizing total earliness criterion also maximizes the average earliness criterion. This is a MAX-SUM problem

Average weighted earliness

$$(wE_{avg}) = \frac{1}{n}\sum_{i=1}^{n} w_i E_i = \frac{1}{n}\sum_{i=1}^{n} w_i d_i - \frac{1}{n}\sum_{i=1}^{n} w_i C_i$$

The average weighted earliness is total weighted earliness divided by the number of jobs. A common problem is to maximize the average weighted earliness. Maximizing total weighted earliness also maximizes the average weighted earliness. This is a MAX-SUM problem.

Maximum earliness (E_{max}) = max (E_1 , E_2 , …, E_n)
E_{max} = max {(d_1 - C_1), (d_2 - C_2), …, (d_n - C_n)}

The maximum earliness (E_{max}) is the longest of the earliness of the jobs. A common problem of interest is to maximize E_{max}. This leads to a MAX- MAX problem.

Performance measures based on number of early jobs: A job is said to be early if it completes before its due date. Scheduling criteria based on number of early are listed below.

$$\text{Let } U_i = \begin{Bmatrix} 1; if\ C_i > d_i \\ 0; otherwise \end{Bmatrix}$$

$$\text{Number of early jobs (NE)} = \sum_{i=1}^{n} (1 - U_i)$$

The number of early jobs measures the number of jobs (n) that are completed before their due dates. Typical problem of interest is to maximize the number of early jobs. Maximizing number of early jobs criterion is equivalent to minimizing number of tardy jobs criterion.

Average number of early jobs

$$(NE_{avg}) = \frac{1}{n} \sum_{i=1}^{n} (1 - U_i)$$

The average number of early jobs is the number of early jobs divided by the number of jobs (n). Typical problem of interest is to maximize the average number of early jobs. Maximizing the average number of early jobs criterion is equivalent to minimizing the average number of tardy jobs criterion.

CONCLUSIONS

About 29 distinct scheduling objectives have been discussed and the expressions for computing their values have been formulated from the first principle. The objectives are all expressed in terms of the completion time of jobs and the given parameters thereby simplifying their use.

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