
q-addix

User Manual

v1.01

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1 Installation

1.1 System requirements

- Microsoft Windows XP or later
- Microsoft Excel 2007 or later

1.2 Installing the software

1. Download the **q-addix** ZIP file.
2. Change to the directory where you downloaded the ZIP file.
3. Unzip the ZIP file using your favorite file compression utility.

1.3 Loading the **q-addix** add-in in Microsoft Excel

1. Start Microsoft Excel.
2. Open **q-addix.xlam**.

Hint: The file **q-addix.xlam** is located in the **q-addix** installation folder.

2 Tutorial

This tutorial is intended to provide you with an overview of the main features of q-addix.

2.1 Life Tables

q-addix has a number of life tables built into it that can be employed to calculate actuarial present values. Throughout this tutorial, we will use the most simple built-in life table that is given by de Moivre's law

$$q_x = \frac{1}{86 - x} \quad (0 \leq x < 86). \quad (1)$$

Refer to section 4 for a full list of built-in life tables.

The first example shows how to compute the curtate future life time assuming de Moivre's law.

Example 1 Input:

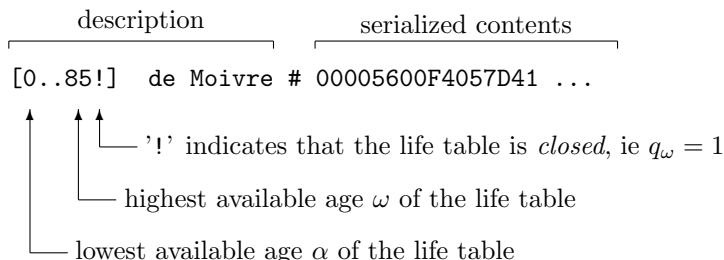
	A
1	=LifeTable("deMoivre")
2	=e_x(A1; 30)

Output:

1	[0..85!] de Moivre # 00005600F4057D41 ...
2	27.5

Let's look at the example more closely: In cell A1, the `LifeTable` function returns a life table that is made available in the background by q-addix. The first argument of the `LifeTable` function is always required and must contain the identifier of the life table to be used for the computation. Depending on the particular life table additional arguments like a person's sex or year of birth must be specified.

The `LifeTable` function returns a string that consists of a description and the serialized contents of the life table.



In cell A2, the the life table is passed to the `e_x` function as the first argument. The second argument is simply the age to be used for the computation. From the output, we see that a life aged 30 has a curtate future lifetime of 27.5 years if we assume de Moivre's law.

Here is an example how to compute q_x and p_x in q-addix. We use again the life table from cell A1.

Example 2 Input:

	A
3	=q_x(A1; 30)
4	=p_x(A1; 30)

Output:

3	0.017857143
4	0.982142857

From (1) we check immediately $q_{30} = \frac{1}{86 - 30} = 0.01785714$.

User defined life tables can be created by the `DefLifeTable` function. See 3.9 for details.

2.2 Actuarial Present Values

q-addix provides a basic set of functions to calculate actuarial present values. In the following example, we calculate the net single premium A_x for a whole life insurance of 1000 payable at the end of the year of death. We anticipate an annual interest rate of 4 %.

Example 3 Input:

	A
5	=1000 * Ax(4%; A1; 30)

Output: 5 396.7824894

In cell A5, the **Ax** function is passed three arguments: the interest rate of 4 %, a reference to cell A1 containing the life table from example 1 and the age of a life which is 30 here. From the output, we see that the net single premium is about 396.78.

We continue example 3 and calculate a net annual premium. If we assume a 20-payment life policy the net annual premium P is obtained by $P = 1000 A_x / \ddot{a}_{x:\bar{t}}$ with $x = 30$ and $t = 20$.

Example 4 Input:

	A
6	=A5 / ä_xn(4%; A1; 30; 20)

Output: 6 32.89518373

In cell A7, the **ä_xn** function is called to calculate the annuity due $\ddot{a}_{x:\bar{t}}$. If you have problems typing the umlaut letter ä on your system you can use the **æ_xn** function that does the same as the **ä_xn** function. Alternatively hold down **Alt** and type **0** **2** **2** **8** on the numeric keypad to get an ä.

Refer to the reference section 3 for detailed descriptions of all functions that can be used to calculate actuarial present values in q-addix.

2.3 Multiple Life Insurances

The following example shows how the **LTJoin** function of q-addix can be employed for computing actuarial present values if two or more persons are involved. We will compute \ddot{a}_{xy} and A_{xy} with $x = 35$ and $y = 30$.

Example 5 Input:

	A
7	=LTJoin(A1; A1; 30 - 35)
8	=ä_x(4%; A7; 35)
9	=Ax(4%; A7; 35)

Output: 7 [5..85!] LTJoin(y=x-5) # ...

8	11.8312457
9	0.541147987

In cell A12, the **LTJoin** function is passed three arguments: the two life tables and the difference $\delta = y - x$. The **LTJoin** function returns a joint life table that is built in the background by q-addix. If $l_z^{(i)}$ denotes the number of people alive at age z in the life table for the i -th person, the numbers of people alive in the joint life table are given by

$$l_z^{(1:2)}(\delta) = l_z^{(1)} \cdot l_{z+\delta}^{(2)}.$$

A joint life table can be used in the same way as any other life table. In order to compute \ddot{a}_{xy} and A_{xy} , we use the already known functions **ä_x** and **Ax** that are passed the joint life table.

3 Function Reference

Throughout the reference section the following notations and conventions are used:

\mathbb{Z}	the integers
$\llbracket a, +\infty \rrbracket$	the set of all integers x satisfying $a \leq x$
$\llbracket a, b \rrbracket$	the set of all integers x satisfying $a \leq x \leq b$
i	the annual effective interest rate
v	the discount factor: $1/(1+i)$
d	the annual effective discount rate: $d = 1 - v$
$i^{(m)}$	the nominal interest rate convertible m times a year: $m((1+i)^{1/m} - 1)$
$d^{(m)}$	the nominal rate of discount convertible m times a year: $m(1 - (1+i)^{-1/m})$
l_x	the number of people alive at age x
q_x	the probability of death between the ages of x and $x+1$
p_x	the probability of a life age x surviving to age $x+1$
T	a life table that is given as a tuple $T = ((l_x)_{\alpha \leq x \leq \omega+1}, (q_x)_{\alpha \leq x \leq \omega})$ with $l_\alpha = 1$. q_x and l_x are connected by the relation

$$q_x = \frac{l_x - l_{x+1}}{l_x} \quad (\alpha \leq x \leq \omega).$$

T is called *closed* if $l_{\omega+1} = 0$.

3.1 \ddot{a}_n

SYNOPSIS Returns the present value $\ddot{a}_{\bar{n}}^{(m)}$ of an annuity-due.
 $\ddot{a}_n(i; n; m)$
 $\text{ae_n}(i; n; m)$

i $[0, +\infty)$ an interest rate
 n $\llbracket 0, +\infty)$ the duration of the annuity
 m $\llbracket 1, +\infty)$ (*optional*) the number of payments per year.
If m is omitted, $m = 1$ is used by default.

DESCRIPTION
$$\ddot{a}_{\bar{n}}^{(m)} = \frac{1}{m} \sum_{k=0}^{nm-1} v^{k/m}$$

EXAMPLE Calculate $\ddot{a}_{\overline{10}}^{(12)}$:

Input:	A
	1 = $\ddot{a}_n(5\%; 10; 12)$
Output:	1 7.929306444

3.2 a_n

SYNOPSIS Returns the present value $a_{\bar{n}}^{(m)}$ of an immediate annuity.
 $a_n(i; n; m)$

i $[0, +\infty)$ an interest rate
 n $\llbracket 0, +\infty)$ the duration of the annuity
 m $\llbracket 1, +\infty)$ (*optional*) the number of payments per year.
If m is omitted, $m = 1$ is used by default.

DESCRIPTION
$$a_{\bar{n}}^{(m)} = \frac{1}{m} \sum_{k=1}^{nm} v^{k/m}$$

EXAMPLE Calculate $a_{\overline{10}}^{(12)}$:

Input:	A
	1 = $a_n(5\%; 10; 12)$
Output:	1 7.897132548

3.3 Ax

SYNOPSIS Returns the present value A_x of a whole life insurance.

Ax(i; T; x)

i [0, +∞) an interest rate

T a closed life table

x $\llbracket \alpha, \omega \rrbracket$ the age of a life

DESCRIPTION
$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_k p_x q_{x+k}$$

EXAMPLE Calculate A_{50} :

		A
Input:	1	=LifeTable("deMoivre")
	2	=Ax(4%; A1; 50)
		A
Output:	1	[0..85!] deMoivre # ...
	2	0.525230054

3.4 Axn

SYNOPSIS Returns the present value $A_{x:\bar{n}}$ of an endowment.

Axn(i; T; x; n)

i [0, +∞) an interest rate

T a life table

x $\llbracket \alpha, \omega \rrbracket$ the age of a life

n $\llbracket 0, +\infty \rrbracket$ the duration of the endowment

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION
$$A_{x:\bar{n}} = \sum_{k=0}^{n-1} v^{k+1} {}_k p_x q_{x+k} + v^n {}_n p_x$$

EXAMPLE Calculate $A_{50:\bar{25}}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=Axn(4%; A1; 50; 25)
		A
Output:	1	[0..85!] deMoivre # ...
	2	0.548565688

3.5 \ddot{a}_x

SYNOPSIS Returns the present value \ddot{a}_x of a whole life annuity-due.

$\ddot{a}_x(i; T; x)$
 $\text{ae_x}(i; T; x)$

i	[0, $+\infty$)	an interest rate
T		a closed life table
x	$[\alpha, \omega]$	the age of a life

DESCRIPTION $\ddot{a}_x = \sum_{k=0}^{\infty} v^k {}_k p_x$

EXAMPLE Calculate \ddot{a}_{50} :

A		
Input:	1	=LifeTable("deMoivre")
	2	= $\ddot{a}_x(4\%; A1; 50)$
Output:	1	[0..85!] deMoivre # ...
	2	12.34401859

3.6 a_x

SYNOPSIS Returns the present value a_x of an immediate whole life annuity.

$a_x(i; T; x)$

i	[0, $+\infty$)	an interest rate
T		a closed life table
x	$[\alpha, \omega]$	the age of a life

DESCRIPTION $a_x = \sum_{k=1}^{\infty} v^k {}_k p_x$

EXAMPLE Calculate a_{50} :

A		
Input:	1	=LifeTable("deMoivre")
	2	= $a_x(4\%; A1; 50)$
Output:	1	[0..85!] deMoivre # ...
	2	11.34401859

3.7 \ddot{a}_{xn}

SYNOPSIS Returns the present value $\ddot{a}_{x:\bar{n}}$ of a temporary life annuity-due.

$\ddot{a}_{xn}(i; T; x; n)$

$a_{xn}(i; T; x; n)$

i $[0, +\infty)$ an interest rate

T a life table

x $[\alpha, \omega]$ the age of a life

n $[0, +\infty)$ the duration of the annuity

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION
$$\ddot{a}_{x:\bar{n}} = \sum_{k=0}^{n-1} v^k k p_x$$

EXAMPLE Calculate $\ddot{a}_{50:\bar{10}}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=a_xn(4%; A1; 50; 10)
Output:	1	[0..85!] deMoivre # ...
	2	7.456536989

3.8 a_{xn}

SYNOPSIS Returns the present value $a_{x:\bar{n}}$ of a temporary immediate life annuity.

$a_{xn}(i; T; x; n)$

i $[0, +\infty)$ an interest rate

T a life table

x $[\alpha, \omega]$ the age of a life

n $[0, +\infty)$ the duration of the annuity

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION
$$a_{x:\bar{n}} = \sum_{k=1}^n v^k k p_x$$

EXAMPLE Calculate $a_{50:\bar{10}}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=a_xn(4%; A1; 50; 10)
Output:	1	[0..85!] deMoivre # ...
	2	6.944444444

3.9 DefLifeTable

SYNOPSIS Creates a user defined life table.

```
DefLifeTable(name; alpha; qx)
    name          the name of the user defined life table
    alpha        [[0,∞)   the lowest available age
    qx            a range containing probabilities
```

EXAMPLE Create a user defined life table with probabilities
 $q_{10} = 0.25$, $q_{11} = 0.50$, $q_{12} = 0.75$ and $q_{13} = 1$.

Input:

	A
1	0.25
2	0.50
3	0.75
4	1
5	=DefLifeTable("foo"; 10; A1:A4)
6	=LTAlpha(A5)
7	=LTOmega(A5)
8	=qx(A5; 10)
9	=qx(A5; 12)

Output:

5	[10..13!] foo # ...
6	10
7	13
8	0.25
9	0.75

3.10 DiscountFactor

SYNOPSIS Returns the discount factor v .

DiscountFactor(i)

i $[0, +\infty)$ an interest rate

DESCRIPTION $v = \frac{1}{1+i}$

EXAMPLE Calculate v with $i = 4\%$:

Input:

	A
1	=DiscountFactor(4%)

Output:

1	0.961538462
---	-------------

3.11 d_x

SYNOPSIS Returns the number ${}_t d_x$ of people who die between age x and age $x + t$.

d_x(T; x; t)

T a life table

x $[\alpha, \omega]$ the age of a life

t $[1, +\infty)$ (*optional*) a duration

If t is omitted, t = 1 is used by default.

$x + t \leq \omega + 1$ is required if T is not closed.

DESCRIPTION ${}_t d_x = l_x - l_{x+t}$

EXAMPLE Calculate d_0 , d_1 and ${}_2 d_0$ where mortality follows de Moivre's law:

A	
1	=LifeTable("deMoivre")
2	=d_x(A1; 0)
3	=d_x(A1; 1)
4	=d_x(A1; 0; 2)
1	[0..85!] deMoivre # ...
2	0.011627907
3	0.011627907
4	0.023255814

3.12 e_x

SYNOPSIS Returns the expected curtate future lifetime e_x .

e_x(T; x)

T a closed life table

x $[\alpha, \omega]$ the age of a life

DESCRIPTION $e_x = \sum_{k=1}^{\infty} kp_x$

EXAMPLE Calculate e_0 , e_{50} and e_{85} where mortality follows de Moivre's law:

A	
1	=LifeTable("deMoivre")
2	=e_x(A1; 0)
3	=e_x(A1; 50)
4	=e_x(A1; 85)
1	[0..85!] deMoivre # ...
2	42.5
3	17.5
4	0

3.13 IAx

DESCRIPTION Returns the present value $(IA)_x$ of an increasing whole life insurance.

IAx(i; T; x)

i $[0, +\infty)$ an interest rate

T a closed life table

x $[\alpha, \omega]$ the age of a life

DESCRIPTION $(IA)_x = \sum_{k=0}^{\infty} (k+1) v^{k+1} {}_k p_x q_{x+k}$

EXAMPLE Calculate $(IA)_{50}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=IAx(4%; A1; 50)
Output:	1	[0..85!] deMoivre # ...
	2	7.564263365

3.14 LifeTable

SYNOPSIS Gets a built-in life table.

LifeTable(id; sex; year; key)

id the identifier of the life table to be returned
id is a case-insensitive string.

sex {1, 2} (optional) 1 = male, 2 = female

year $[1, +\infty)$ (optional) year of birth

aid $[0, +\infty)$ (optional) additional key to identify the life table

Refer to section 4 for a full list of built-in life tables. Section 4 describes for each built-in life table which arguments are required by the **LifeTable** function.

EXAMPLE 1

Get the life table that corresponds to de Moivre's law.

		A
Input:	1	=LifeTable("deMoivre")
	2	=LTAlpha(A1)
	3	=LTOmega(A1)
	4	=q_x(A1; 85)
	5	=e_x(A1; 0)
Output:	1	[0..85!] deMoivre # ...
	2	0
	3	85
	4	1
	5	42.5

EXAMPLE 2 Get a real world life table that is currently used for the calculation of whole life insurances in Germany.

	A
Input:	1 =LifeTable("de.DAV.2008T.Aggregat"; 1)
	2 =LTAlpha(A1)
	3 =LTOmega(A1)
	4 =q_x(A1; 85)
	5 =e_x(A1; 0)
Output:	1 [0..119!] de.DAV.2008T.Aggregat M # ...
	2 0
	3 119
	4 0.1665
	5 73.97968717

3.15 LTAdd

SYNOPSIS Adds two life tables.

LTAdd(T1; T2)

T1 a life table

T2 a life table

DESCRIPTION Let T_j be given by

$$T_j = \left((l_x^{(j)})_{\alpha_j \leq x \leq \omega_j + 1}, (q_x^{(j)})_{\alpha_j \leq x \leq \omega_j} \right) \quad (j = 1, 2).$$

The probabilities of the resulting life table are given by

$$q_x^{(1)} + q_x^{(2)} - q_x^{(1)} q_x^{(2)}$$

with $\min(\alpha_1, \alpha_2) \leq x \leq \max(\omega_1, \omega_2)$ and assuming $q_x^{(j)} = 0$ for $j \notin [\alpha_j, \omega_j]$.

EXAMPLE Add two tables and calculate e_x for $x = 0$.

	A
Input:	1 =LifeTable("deMoivre")
	2 =LTAdd(A1; A1)
	2 =e_x(A2; 0)
Output:	1 [0..85!] deMoivre # ...
	2 [0..85!] LTAdd() # ...
	3 28.16860465

3.16 LTAlpha

SYNOPSIS Returns the lowest available age of a life table.

LTAlpha(T)

T	a life table
---	--------------

EXAMPLE Get the lowest available age of de Moivre's table:

Input:	A
	1 =LifeTable("deMoivre") 2 =LTAlpha(A1)
Output:	1 [0..85!] deMoivre # ... 2 0

3.17 LTJoin

SYNOPSIS Combines two life tables to get a life table for the joint life status.

JoinLifeTables(T1; T2; delta)

T1	a life table	
T2	a life table	
delta	\mathbb{Z}	an age difference

DESCRIPTION Let T_j be given by

$$T_j = \left((l_x^{(j)})_{\alpha_j \leq x \leq \omega_j + 1}, (q_x^{(j)})_{\alpha_j \leq x \leq \omega_j} \right) \quad (j = 1, 2).$$

The probabilities of the resulting life table are given by

$$\frac{l_x^{(1)} l_{x+\delta}^{(2)} - l_{x+1}^{(1)} l_{x+\delta+1}^{(2)}}{l_x^{(1)} l_{x+\delta}^{(2)}}$$

with $\max(\alpha_1, \alpha_2 - \delta) \leq x \leq \min(\omega_1, \omega_2 - \delta)$.

EXAMPLE Calculate \ddot{a}_{xy} with $x = 50$ and $y = 45$ assuming de Moivre's law.

Input:	A
	1 =LifeTable("deMoivre") 2 =LTJoin(A1; A1; 45 - 50) 2 =ä_x(4%; A2; 50)
Output:	1 [0..85!] deMoivre # ... 2 [5..85!] LTJoin(y=x-5) # ... 3 9.589045824

3.18 LTMult

SYNOPSIS Multiplies a life table with a factor.

LTMult(c; T)

c $(0, +\infty)$ a factor

T a life table

DESCRIPTION The probabilities of the resulting life table are given by

$$\begin{cases} \min(1, c \cdot q_x) & q_x < 1 \\ 1 & q_x = 1 \end{cases}$$

with $\alpha \leq x \leq \omega$.

EXAMPLE Calculate e_x with $x = 0$ assuming twice the mortality of de Moivre's law.

		A
Input:	1	=LifeTable("deMoivre")
	2	=LTMult(2; A1)
	2	=e_x(A2; 0)
Output:		
	1	[0..85!] deMoivre # ...
	2	[0..84!] LTMult(2) # ...
	3	28

3.19 LTOMega

SYNOPSIS Returns the highest available age of a life table.

LTOMega(T)

T a life table

EXAMPLE Get the highest available age of de Moivre's table:

		A
Input:	1	=LifeTable("deMoivre")
	2	=LTOMega(A1)
Output:		
	1	[0..85!] deMoivre # ...
	2	85

3.20 l_x

SYNOPSIS Returns the number of people alive at age x .

l_x(T; x)

T a life table

x $\llbracket \alpha, +\infty \rrbracket$ the age of a life

$x \leq \omega + 1$ is required if T is not closed.

EXAMPLE Calculate l_0 , l_1 , l_{85} and l_{86} for de Moivre's table:

A	
1	=LifeTable("deMoivre")
2	=l_x(A1; 0)
3	=l_x(A1; 1)
4	=l_x(A1; 85)
5	=l_x(A1; 86)

A	
1	[0..85!] deMoivre # ...
2	1
3	0.988372093
4	0.011627907
5	0

3.21 nAx

DESCRIPTION Returns the present value $|_n A_x$ of a term life insurance.

nAx(i; T; x; n)

i $[0, +\infty)$ an interest rate

T a life table

x $\llbracket \alpha, \omega \rrbracket$ the age of a life

n $\llbracket 0, +\infty \rrbracket$ the duration of the term life insurance

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION $|_n A_x = \sum_{k=0}^{n-1} v^{k+1} k p_x q_{x+k}$

EXAMPLE Calculate $|_{10} A_{50}$:

A	
1	=LifeTable("deMoivre")
2	=nAx(4%; A2; 50; 10)

A	
1	[0..85!] deMoivre # ...
2	0.225302661

3.22 nEx

SYNOPSIS Returns the present value of a pure endowment.

nEx(i; T; x; n)

i $[0, +\infty)$ an interest rate

T a life table

x $[\alpha, \omega]$ the age of a life

n $[0, +\infty)$ the duration of the pure endowment

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION $_nE_x = v^n \ _n p_x$.

EXAMPLE Calculate ${}_{10}E_{50}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=nEx(4%; A1; 50; 10)
Output:		
	1	[0..85!] deMoivre # ...
	2	0.487907455

3.23 nIAx

SYNOPSIS Returns the present value ${}_n(IA)_x$ of an increasing term life insurance.

nIAx(i; T; x; n)

i $[0, +\infty)$ an interest rate

T a life table

x $[\alpha, \omega]$ the age of a life

n $[0, +\infty)$ the duration of the term life insurance

$x + n \leq \omega + 1$ is required if T is not closed.

DESCRIPTION ${}_n(IA)_x = \sum_{k=0}^{n-1} (k+1) v^{k+1} {}_k p_x q_{x+k}$

EXAMPLE Calculate ${}_{10}(IA)_{50}$:

		A
Input:	1	=LifeTable("deMoivre")
	2	=nIAx(4%; A1; 50; 10)
Output:		
	1	[0..85!] deMoivre # ...
	2	1.166451335

3.24 NominalDiscountRate

SYNOPSIS Returns the annual nominal rate of discount $d^{(m)}$ convertible m times a year.

NominalDiscountRate(i; m)

i $[0, +\infty)$ the annual effective interest rate

m $\llbracket 1, +\infty \rrbracket$ the number of payments per year

DESCRIPTION $d^{(m)} = m \left(1 - (1 + i)^{-1/m} \right)$

EXAMPLE Calculate $d^{(12)}$ with $i = 5\%$:

Input:	<input type="text" value="A"/>
	<input type="text" value="1 =NominalDiscountRate(5%; 12)"/>
Output:	<input type="text" value="1 0.048691112"/>

3.25 NominalInterestRate

SYNOPSIS Returns the annual nominal interest rate $i^{(m)}$ convertible m times a year:

NominalInterestRate(i; m)

i $[0, +\infty)$ the annual effective interest rate

m $\llbracket 1, +\infty \rrbracket$ the number of payments per year

DESCRIPTION $i^{(m)} = m \left((1 + i)^{1/m} - 1 \right)$

EXAMPLE Calculate $i^{(12)}$ with $i = 5\%$:

Input:	<input type="text" value="A"/>
	<input type="text" value="1 =NominalInterestRate(5%; 12)"/>
Output:	<input type="text" value="1 0.048889485"/>

3.26 p_x

SYNOPSIS Returns the probability ${}_t p_x$ of a life age x surviving to age $x + t$.

p_x(T; x; t)

T a life table

x $\llbracket \alpha, \omega \rrbracket$ the age of a life

t $\llbracket 1, +\infty \rrbracket$ (*optional*) a duration

If *t* is omitted, *t* = 1 is used by default.

$x + t \leq \omega + 1$ is required if *T* is not closed.

DESCRIPTION ${}_t p_x = \frac{l_{x+t}}{l_x}$

EXAMPLE Calculate p_{50} , $25p_{50}$ and $50p_{50}$ where mortality follows de Moivre's law:

	A
Input:	1 =LifeTable("deMoivre") 2 =p_x(A2; 50) 3 =p_x(A2; 50; 25) 4 =p_x(A2; 50; 50)
Output:	1 [0..85!] deMoivre # ... 2 0.972222222 3 0.305555556 4 0

3.27 q_x

SYNOPSIS Returns the probability tq_x of death between the ages of x and $x + t$.

$q_x(T; x; t)$

T a life table

x $[\alpha, \omega]$ the age of a life

t $[1, +\infty)$ (*optional*) a duration

If t is omitted, $t = 1$ is used by default.

$x + t \leq \omega + 1$ is required if T is not closed.

DESCRIPTION
$$tq_x = \frac{l_x - l_{x+t}}{l_x}$$

EXAMPLE Calculate q_{50} , $25q_{50}$ and $50q_{50}$ where mortality follows de Moivre's law:

	A
Input:	1 =LifeTable("deMoivre") 2 =q_x(A2; 50) 3 =q_x(A2; 50; 25) 4 =q_x(A2; 50; 50)
Output:	1 [0..85!] deMoivre # ... 2 0.02777778 3 0,694444444 4 1

3.28 XInfo

DESCRIPTION Returns the product version of the q-addix add-in.

EXAMPLE

	A
Input:	1 =XInfo()
Output:	1 q-addix v1.01 ...

4 Life Table Reference

4.1 deMoivre

SYNOPSIS Mortalities $q_x = \frac{1}{86 - x}$ from de Moivre's law.

id	sex	year	aid
"deMoivre"	ignored	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("deMoivre") 2 =qx(A1; 30)
Output:	1 [0..85!] deMoivre # ... 2 0.017857143

4.2 de.DAV.1994R

SYNOPSIS Mortalities from [8].

id	sex	year	aid
"de.DAV.1994R"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1994R"; 1; 1980) 2 =qx(A1; 30)
Output:	1 [3..114!] de.DAV.1994R M:[1974, 1981] # ... 2 0.000814

4.3 de.DAV.1994T

SYNOPSIS Mortalities from [7].

id	sex	year	aid
"de.DAV.1994T"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1994T"; 2) 2 =qx(A1; 30)
Output:	1 [0..101!] de.DAV.1994T F # ... 2 0.000689

4.4 de.DAV.1997I

SYNOPSIS Probabilities of becoming disabled from [6].

id	sex	year	aid
"de.DAV.1997I"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1997I"; 1) 2 =qx(A1; 30)
Output:	1 [15..69] de.DAV.1997I M # ... 2 0.0022807

4.5 de.DAV.1997RI

SYNOPSIS Probabilities of reversing disability from [6].

id	sex	year	aid
"de.DAV.1997RI"	required	ignored	required

aid holds the age of becoming disabled.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1997RI"; 1; 0; 25) 2 =qx(A1; 30)
Output:	1 [25..69] de.DAV.1997RI M 25 # ... 2 0.074545

4.6 de.DAV.1997TI

SYNOPSIS Probabilities of dying disabled from [6].

id	sex	year	aid
"de.DAV.1997TI"	required	ignored	required

aid holds the age of becoming disabled.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1997TI"; 1; 0; 25) 2 =qx(A1; 30)
Output:	1 [25..69] de.DAV.1997TI M 25 # ... 2 0.0038612

4.7 de.DAV.1998E

SYNOPSIS Probabilities of becoming disabled from [1].

id	sex	year	aid
"de.DAV.1998E"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1998E"; 2) 2 =qx(A1; 30)
Output:	1 [15..69] de.DAV.1998E F # ... 2 0.0008043

4.8 de.DAV.1998RE

SYNOPSIS Probabilities of reversing disability from [1].

id	sex	year	aid
"de.DAV.1998RE"	required	ignored	required

aid holds the age of becoming disabled.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1998RE"; 2; 0; 25) 2 =qx(A1; 30)
Output:	1 [25..69] de.DAV.1998RE F 25 # ... 2 0.0372946

4.9 de.DAV.1998TE

SYNOPSIS Probabilities of dying disabled from [1].

id	sex	year	aid
"de.DAV.1998TE"	required	ignored	required

aid holds the age of becoming disabled.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.1998TE"; 2; 0; 25) 2 =qx(A1; 30)
Output:	1 [25..69] de.DAV.1998TE F 25 # ... 2 0.0028827

4.10 de.DAV.2004R.Aggregat

SYNOPSIS Mortalities from [2].

id	sex	year	aid
"de.DAV.2004R.Aggregat"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R.Aggregat"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R.Aggregat M:1980 # ...
	2 0.003310851

4.11 de.DAV.2004R.Selektion

SYNOPSIS Mortalities from [2].

id	sex	year	aid
"de.DAV.2004R.Selektion"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R.Selektion"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R.Selektion M:1980 # ...
	2 0.003679332

4.12 de.DAV.2004R.Selektion/Faktoren

SYNOPSIS Mortalities from [2]. During the first five years after retirement, mortalities are calculated using the selection factors f^1 and f^{2-5} .

id	sex	year	aid
"de.DAV.2004R.Selektion/Faktoren"	required	required	required

aid holds the retirement age.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R.Selektion/Faktoren"; 1; 1980; 65)
	2 =qx(A1; 68)
Output:	1 [65..121!] de.DAV.2004R.Selektion/Faktoren M:1980 65 # ...
	2 0.003223864

4.13 de.DAV.2004R-B.Aggregat

SYNOPSIS Mortalities from [3].

id	sex	year	aid
"de.DAV.2004R-B.Aggregat"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B.Aggregat"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R-B.Aggregat M:1980 # ...
	2 0.005852251

4.14 de.DAV.2004R-B.Selektion

SYNOPSIS Mortalities from [3].

id	sex	year	aid
"de.DAV.2004R-B.Selektion"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B.Selektion"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R-B.Selektion M:1980 # ...
	2 0.006503608

4.15 de.DAV.2004R-B.Selektion/Faktoren

SYNOPSIS Mortalities from [3]. During the first five years after retirement, mortalities are calculated using the selection factors f^1 and f^{2-5} .

id	sex	year	aid
"de.DAV.2004R-B.Selektion/Faktoren"	required	required	required

aid holds the retirement age.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B.Selektion/Faktoren"; 1; 1980; 65)
	2 =qx(A1; 68)
Output:	1 [65..121!] de.DAV.2004R-B.Selektion/Faktoren M:1980 65 # ...
	2 0.00569852

4.16 de.DAV.2004R-B20.Aggregat

SYNOPSIS Mortalities from [3].

id	sex	year	aid
"de.DAV.2004R-B20.Aggregat"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B20.Aggregat"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R-B20.Aggregat M:1980 # ...
	2 0.004119669

4.17 de.DAV.2004R-B20.Selektion

SYNOPSIS Mortalities from [3].

id	sex	year	aid
"de.DAV.2004R-B20.Selektion"	required	required	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B20.Selektion"; 1; 1980)
	2 =qx(A1; 68)
Output:	1 [0..121!] de.DAV.2004R-B20.Selektion M:1980 # ...
	2 0.00457819

4.18 de.DAV.2004R-B20.Selektion/Faktoren

SYNOPSIS Mortalities from [3]. During the first five years after retirement, mortalities are calculated using the selection factors f^1 and f^{2-5} .

id	sex	year	aid
"de.DAV.2004R-B20.Selektion/Faktoren"	required	required	required

aid holds the retirement age.

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2004R-B20.Selektion/Faktoren"; 1; 1980; 65)
	2 =qx(A1; 30)
Output:	1 [65..121!] de.DAV.2004R-B20.Selektion/Faktoren M:1980 65 # ...
	2 0.004011451

4.19 de.DAV.2004T.Aggregat

SYNOPSIS Mortalities from [4].

id	sex	year	aid
"de.DAV.2008T.Aggregat"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2008T.Aggregat"; 2)
	2 =qx(A1; 30)
Output:	1 [0..120!] de.DAV.2008T.Aggregat F # ...
	2 0.000311

4.20 de.DAV.2004T.Nichtraucher

SYNOPSIS Mortalities from [5].

id	sex	year	aid
"de.DAV.2008T.Nichtraucher"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2008T.Nichtraucher"; 2)
	2 =qx(A1; 30)
Output:	1 [0..120!] de.DAV.2008T.Nichtraucher F # ...
	2 0.000283

4.21 de.DAV.2004T.Raucher

SYNOPSIS Mortalities from [5].

id	sex	year	aid
"de.DAV.2008T.Raucher"	required	ignored	ignored

EXAMPLE

	A
Input:	1 =LifeTable("de.DAV.2008T.Raucher"; 2)
	2 =qx(A1; 30)
Output:	1 [0..117!] de.DAV.2008T.Raucher F # ...
	2 0.000531

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